Optical Networks: A Practical Perspective
Third Edition

Solutions Manual for Instructors

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Preface

In many cases, the problems in the book require further exploration of the topics in detail as opposed to simply plugging numbers into equations. Instructors may therefore want to review the solutions before assigning problems. See the book's web page *http://www.elsevierdirect.com/9780123740922* for the current errata of the book, as well as this solutions manual. If you discover an error that is not listed there, we would very much appreciate your letting us know about it. You can email *rajivramaswami@ieee.org*, or *kumar@tejasnetworks.com* or *galens@hawaii.edu*.

Note that all equation and figure numbers used in this manual refer to those in the third edition of the book.

2 chapter

Propagation of Signals in Optical Fiber

2.1 From Snell's Law we have,

$$n_0 \sin\theta_0^{\max} = n_1 \sin\theta_1^{\max}.$$

Using the definition of $\theta_0^{\max}$ from Figure 2.3, we have

$$n_1 \sin\left(\pi/2 - \theta_1^{\max}\right) = n_2,$$

or,

$$n_1 \cos\theta_1^{\max} = n_2,$$

or,

$$\sin\theta_1^{\max} = \sqrt{1 - \frac{n_2^2}{n_1^2}}.$$

Therefore,

$$n_0 \sin\theta_0^{\max} = n_1 \sqrt{\frac{1 - n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

which is (2.2).

2.2 From (2.2),

$$\frac{\delta T}{L} = \frac{1}{c}\frac{n_1^2}{n_2}\Delta = 10 \text{ ns/km}.$$

Therefore,

$$n_1^2\Delta = \frac{n_2 c\delta T}{L}.$$

We have,

$$NA = n_1\sqrt{2\Delta} = \sqrt{2n_2 c\delta T/L} = \sqrt{2 \times 1.45 \times 3 \times 10^5 \times 10^{-8}} = 0.093.$$

The maximum bit rate is given by

$$\frac{0.5}{(10 \text{ ns/km} \times 20 \text{ km})} = 2.5 \text{ Mb/s}.$$

2.3 We have

$$\nabla \times H = J + \frac{\partial D}{\partial t}.$$

Using $J = 0$ and taking the curl of both sides, we get

$$\nabla \times \nabla \times H = \frac{\partial(\nabla \times D)}{\partial t} = \frac{\partial}{\partial t}\, \epsilon_0(\nabla \times E) + \frac{\partial(\nabla \times P)}{\partial t}.$$

Here we have used the relation $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$. Using (2.13), this simplifies to

$$\nabla \times \nabla \times H = -\epsilon_0\frac{\partial^2 B}{\partial t^2} + \frac{\partial(\nabla \times P)}{\partial t}.$$

Taking Fourier transforms, we have

$$\begin{aligned}
\nabla \times \nabla \times \tilde{H} &= \epsilon_0\omega^2\tilde{B} - i\omega(\nabla \times \tilde{P}) \\
&= \epsilon_0\omega^2\mu_0\tilde{H} - i\omega\epsilon_0\tilde{\chi}(\nabla \times \tilde{E}) \\
&= \epsilon_0\omega^2\mu_0\tilde{H} - i\omega\epsilon_0\tilde{\chi}(i\omega\mu_0\tilde{H}) \\
&= \epsilon_0\mu_0\omega^2(1 + \tilde{\chi})\tilde{H} = \epsilon_0\mu_0\omega^2 n^2(\omega)\tilde{H} \\
&= \frac{\omega^2 n^2}{c^2}\tilde{H}.
\end{aligned}$$

Using $\nabla \times \nabla \times \tilde{H} = \nabla(\nabla \cdot \tilde{H}) - \nabla^2\tilde{H}$, we get

$$\nabla^2\tilde{H} + \frac{\omega^2 n^2}{c^2}\,\tilde{H} = \nabla(\nabla \cdot \tilde{H}) = 0, \text{ since } \nabla \cdot B = 0.$$

2.4 Using $\frac{2\pi}{\lambda}\, a\sqrt{n_1^2 - n_2^2} < 2.405$,

$$\lambda_{\text{cutoff}} = \frac{2\pi a}{2.405}\,\sqrt{n_1^2 - n_1^2} \approx \frac{2\pi a}{2.405}\, n_1\sqrt{2\Delta}.$$

For $a = 4\ \mu$m and $\Delta = 0.003$, $\lambda_{\text{cutoff}} = 1.214\ \mu$m, assuming $n_1 = 1.5$.

2.5 (a) We have

$$\lambda_{\text{cutoff}} = \frac{2\pi a}{2.405}\sqrt{n_1^2 - n_2^2}.$$

Using $a = 4\ \mu$m, $n_2 = 1.45$, and $\lambda_{\text{cutoff}} = 1.2\ \mu$m yields

$$n_1 = \sqrt{\left(\frac{2.405 \times 1.2}{2 \times \pi \times 4}\right)^2 + 1.45^2} = 1.45454.$$

Therefore $1.45 < n_1 < 1.45454$ for the fiber to be single moded for $\lambda > 1.2\ \mu$m.

(b) We have

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}.$$

Using $a = 4\ \mu$m, $\lambda = 1.55\ \mu$m, $n_2 = 1.45$ and $V = 2.0$, we have

$$n_1 = \sqrt{\left(\frac{V\lambda}{2\pi a}\right)^2 + n_2^2} = 1.4552.$$

Using

$$b(V) \approx \left(1.1428 - \frac{0.9960}{V}\right)^2,$$

we obtain $b(2.0) = 0.41576$. We also have

$$b = \frac{n_{\text{eff}}^2 - n_2^2}{n_2^2 - n_2^2}.$$

Therefore, we can calculate $n_{\text{eff}} = 1.45218$. Thus

$$\beta = \frac{2\pi n_{eff}}{\lambda} = 5.887\ /\mu\text{m}.$$

2.6 The specified nominal value of a must satisfy

$$\lambda_{\text{cutoff}} < \frac{2\pi(1.05a)}{2.405}\, n_1\sqrt{2 \times 1.1 \times 0.005}$$

for $\lambda_{\text{cutoff}} = 1.2\mu m$ and $n_1 = 1.5$. Thus the largest value that can be specified is

$$a = \frac{1.2 \times 2.405}{2\pi \times 1.05 \times 1.5 \times \sqrt{2 \times 1.1 \times 0.005}} = 2.78\ \mu\text{m}.$$

Note that we have used the property that λ_{cutoff} increases with increase in a or Δ so that the largest possible values of a and Δ are used in calculating the cutoff wavelength.

2.7 We have

$$\frac{\partial A}{\partial z} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2} = 0.$$

Taking Fourier transforms, we get

$$\begin{aligned}\frac{\partial \tilde{A}}{\partial z} &+ \frac{i}{2}\beta_2\,(-i\omega)^2\tilde{A} = 0, \text{ or,}\\ \frac{\partial \tilde{A}}{\partial z} &- \frac{i\beta_2\omega^2}{2}\tilde{A} = 0.\end{aligned}$$

Solving this for $\tilde{A}(z,\omega)$, we get

$$\tilde{A}(z,\omega) = \tilde{A}(0,\omega)\ \exp\left[\frac{i\beta_2\omega^2}{2}z\right].$$

Note that

$$
\begin{aligned}
\tilde{A}(0,\omega) &= \int_{-\infty}^{\infty} A(0,t)e^{i\omega t}\,dt = \int_{-\infty}^{\infty} A_0 e^{-t^2/2T_0^2}\,e^{i\omega t}dt \\
&= A_0 \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{t^2}{T_0^2}-2i\omega t+(i\omega T_0)^2\right)}\,e^{\frac{1}{2}(i\omega T_0)^2}\,dt \\
&= A_0 e^{-\omega^2 T_0^2/2}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{t}{T_0}-i\omega T_0\right)^2}\,dt \\
&= A_0 e^{-\omega^2 T_0^2/2}\sqrt{2\pi T_0^2}.
\end{aligned}
$$

Therefore,

$$
\tilde{A}(z,\omega) = A_0 T_0\sqrt{2\pi}e^{-\omega^2 T_0^2/2}\,\exp\left(i\frac{i\beta_2\omega^2}{2}z\right).
$$

Using this, we obtain

$$
\begin{aligned}
A(z,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{A}(z,\omega)e^{-i\omega t}\,d\omega = \frac{A_0T_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{\omega^2T_0^2}{2}}\,e^{\frac{i\beta_2\omega^2}{2}z}\,e^{-i\omega t}d\omega \\
&= \frac{A_0T_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left[\omega^2\left(T_0^2-i\beta_2 z\right)+2i\omega t+\frac{(it)^2}{T_0^2-i\beta_2 z}\right]}e^{-t^2/2(T_0^2-i\beta_2 z)}\,d\omega \\
&= \frac{A_0T_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\sqrt{\omega^2\left(T_0^2-i\beta_0 z\right)}+\frac{it}{\sqrt{T_0^2-i\beta z}}\right)^2}\,e^{-t^2/2(T_0^2-i\beta_2 z)}\,d\omega \\
&= \frac{A_0T_0}{\sqrt{2\pi}}\,e^{-\frac{t^2}{2(T_0^2-i\beta_2 z)}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(T_0^2-i\beta_2 z)\left(\omega+\frac{it}{T_0^2-i\beta_2 z}\right)^2}\,d\omega \\
&= A_0T_0e^{-t^2/2(T_0^2-i\beta_2 z)}\frac{1}{\sqrt{T_0^2-i\beta_2 z}}.
\end{aligned}
$$

Note that in the last step we used the formula given in the problem with

$$
\alpha = \frac{1}{T_0-i\beta_2 z} = \frac{T_0+i\beta_2 z}{T_0^2+\beta_2^2 z^2},
$$

with

$$
\mathrm{Re}(\alpha) = \frac{T_0}{T_0^2+\beta_2^2z^2} > 0.
$$

2.8 From (E.8), we derive (E.9) and (E.10) as discussed in Appendix E. (2.13) now follows from (E.10).

2.9 From (2.28), with $\kappa = 0$,

$$A(z,t) = \frac{A_0 T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left(-\frac{1}{2}\frac{(t-\beta_2 z)^2}{(T_0^2 - i\beta_2 z)}\right),$$

which is the envelope of a Gaussian pulse for all z. Letting $t' = t = \beta_1 z$ (so that we choose a reference frame moving with the pulse), we have

$$\begin{aligned} A(z,t') &= \frac{A_0 T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left(-\frac{1}{2}\frac{t'^2}{(T_0^2 - i\beta_2 z)}\right) \\ &= \frac{A_0 T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left(-\frac{1}{2}\frac{t'^2(T_0^2 + i\beta_2 z)}{T_0^4 + (\beta_2 z)^2}\right). \end{aligned}$$

The phase of this pulse is

$$\omega_0 t' + \frac{\beta_2 z t'^2}{2(T_0^4 + (\beta_2 z)^2)}.$$

Hence the chirp factor is, comparing with (2.26),

$$\kappa = \frac{\beta_2 z T_0^2}{T_0^4 + (\beta_2 z)^2} = \frac{\beta_2 z / T_0^2}{1 + \left(\frac{\beta_2 z}{T_0^2}\right)^2} = \frac{\mathrm{sgn}(\beta_2) z / L_D}{1 + \left(\frac{z}{L_D}\right)^2}.$$

2.10 A Gaussian pulse is described by

$$A(t) = A_0 e^{-\frac{1}{2}t^2/T_0^2}.$$

Its *rms* width is given by

$$T^{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |A(t)|^2\, dt}{\int_{-\infty}^{\infty} |A(t)|^2\, dt}}.$$

We have

$$\int_{-\infty}^{\infty} |A(t)|^2 dt = \int_{-\infty}^{\infty} A_0^2 e^{-t^2/T^2}\, dt = \sqrt{2\pi}\,\frac{T_0 A_0^2}{\sqrt{2}} = T_0 \sqrt{\pi} A_0^2,$$

and

$$\int_{-\infty}^{\infty} t^2 |A(t)|^2 dt = A_0^2 \int_{-\infty}^{\infty} t^2 e^{-t^2/T_0^2}\, dt = A_0^2 \frac{T_0}{\sqrt{2}} \sqrt{2\pi}\, \frac{T_0^2}{2}.$$

Therefore,

$$T^{rms} = \sqrt{\frac{T_0^2}{2}} = \frac{T_0}{\sqrt{2}}.$$

2.11 From (2.13),

$$\frac{|T_z|}{T_0} = \sqrt{\left(1 + \frac{\kappa\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}.$$

For positive κ and negative β_2,

$$\frac{|T_z|}{T_0} = \sqrt{\left(1 - \frac{\kappa z}{L_D}\right)^2 + \left(\frac{z}{L_D}\right)^2}.$$

(a) Differentiating the equation above, the minimum pulse width occurs for $z = z_{\min}$ which solves

$$-\kappa\left(1 - \frac{\kappa z}{L_D}\right) + \frac{z}{L_D} = 0.$$

This yields

$$z_{\min} = \frac{\kappa}{1+\kappa^2}\, L_D.$$

For $\kappa = 5$,

$$z_{\min} = \frac{5}{26} L_D = 0.192 L_D.$$

(b) The pulse width equals that of an unchirped pulse if

$$\left(1 - \frac{\kappa z}{L_D}\right)^2 + \left(\frac{z}{L_D}\right)^2 = 1 + \left(\frac{z}{L_D}\right)^2,$$

that is, if

$$z = \frac{2L_D}{\kappa}.$$

For $\kappa = 5$, we get $z = 0.4 L_D$.

2.12 We leave this to the reader to go through the algebra and verify.

2.13 For a first order soliton,

$$N^2 = \frac{\gamma P_0}{|\beta_2|/T_0^2} = 1.$$

Using $\gamma = 1$ /W-km, $\beta_2 = 2$ ps^2/km, and $P_0 = 50$ mW,

$$T_0 = \sqrt{\frac{|\beta_2|}{\gamma P_0}} = 6.32 \text{ ps}.$$

Recall that a soliton pulse is described by

$$\operatorname{sech}\left(\frac{t}{T_0}\right) = \frac{2}{e^{-t/T_0} + e^{t/T_0}}$$

(neglecting the phase and considering a reference frame moving with the pulse). The *half width* at half maximum is given by the solution to

$$\left[\operatorname{sech}\left(\frac{t}{T_0}\right)\right]^2 = \frac{1}{2} \quad \text{or} \quad \operatorname{sech}\frac{t}{T_0} = \frac{1}{\sqrt{2}}.$$

Solving this yields $t = T_0 \ln(\sqrt{2}+1)$. Therefore,

$$T_{FWHM} = [2\ln(\sqrt{2}+1)]\; T_0 = 1.763T_0.$$

Using $T_0 = 6.32$ ps, we get $T_{FWHM} = 11.15$ ps. Therefore,

$$B < \frac{1000}{11.15} \times \frac{1}{10}\ \text{Gb/s} = 8.97\ \text{Gb/s},$$

where we have used the condition that the bit period $> 10 \times T_{FWHM}$.

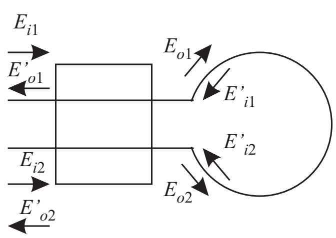
3

chapter

Components

3.1 Using (3.1), we can write

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = e^{-i\beta l} \begin{pmatrix} \cos kl & i \sin kl \\ i \sin kl & \cos kl \end{pmatrix} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}.$$



Note that $kl = \pi/4$ for a 3–dB coupler. Using this and ignoring the common phase factor $e^{-i\beta l}$, we get

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}.$$

The traversal around the loop introduces the same phase change in E_{o1} and E_{o2}, which can be ignored. Thus $E'_{i1} = E_{o2}$ and $E'_{i2} = E_{o1}$.

The directional coupler is a reciprocal device. Therefore, the transfer function is the same if the inputs and outputs are interchanged. Thus

$$\begin{aligned} \begin{pmatrix} E'_{o1} \\ E'_{o2} \end{pmatrix} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E'_{i1} \\ E'_{i2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2i & 0 \\ 0 & 2i \end{pmatrix} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix} = i \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix}. \end{aligned}$$

Thus, $E'_{o1} = i\ E_{i1}$ and $E'_{o2} = i\ E_{i2}$. Therefore, the input field is reflected (with a phase shift) and the device acts as a mirror.

3.2 The scattering matrix is given by

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & s_{13} \\ 0 & 0 & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix}.$$

If the device satisfies the conservation of energy condition, then $\mathbf{S}^T\mathbf{S}^* = I$. In this case, we would then have

$$\begin{pmatrix} 0 & 0 & s_{31} \\ 0 & 0 & s_{32} \\ s_{13} & s_{23} & s_{33} \end{pmatrix} \begin{pmatrix} 0 & 0 & s_{13}^* \\ 0 & 0 & s_{23}^* \\ s_{31}^* & s_{32}^* & s_{33}^* \end{pmatrix} = I.$$

This implies that $|s_{31}|^2 = 1$ and $|s_{32}|^2 = 1$. Also $s_{31}s_{32}^* = 0$, which implies that either $s_{31} = 0$ or $s_{32} = 0$, both of which would contradict the previous condition. Therefore the device cannot satisfy the conservation of energy condition.

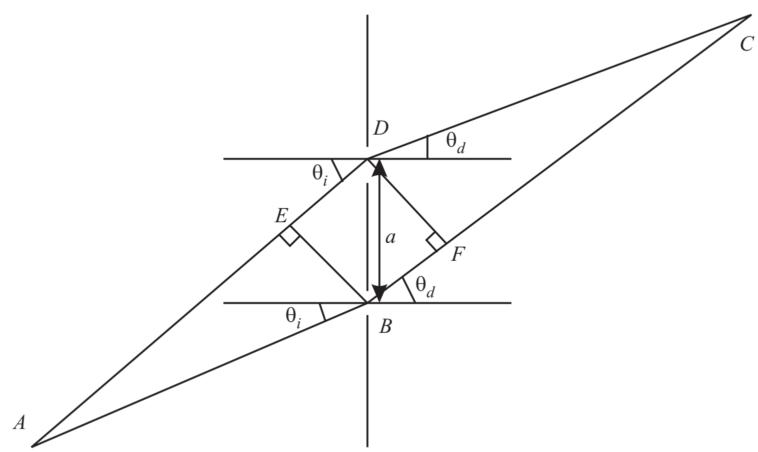
3.3 We have

$$\mathbf{S}^T\mathbf{S}^* = \begin{pmatrix} s_{11} & 0 \\ s_{12} & s_{22} \end{pmatrix} \begin{pmatrix} s_{11}^* & s_{12}^* \\ 0 & s_{22}^* \end{pmatrix}.$$

Assume that $\mathbf{S}^T\mathbf{S}^* = I$. Then $|s_{11}|^2 = 1$ and $s_{11}s_{12}^* = 0$. If $s_{12} \neq 0$, that is, power is transferred from port 1 to port 2, $s_{11} = 0$, which is a contradiction.

3.4 We assume the pitch of the grating, a, is small compared to the distance from the source or imaging plane to the grating plane. Thus the rays from A to both the slits can be taken to be approximately parallel. The same goes for the rays from both the slits to C. Then the difference in the path lengths ADC and ABC is

$$\begin{aligned} ED - BF &\approx a\sin\theta_i - a\sin\theta_d \\ &= a[\sin\theta_i - \sin\theta_d]. \end{aligned}$$



3.5 The grating equation is:

$$d(\sin\theta_i + \sin\theta_d) = N\lambda$$

where θ_i and θ_d are measured with respect to the vertical axis in Figure 3.11. (The blazing angle α is also measured with respect to the same axis.) d is the periodicity of the grating (in the horizontal axis in Figure 3.11).

The derivation is similar to that of the transmission grating in the text (Figure 3.10): The path length difference between rays incident on successive slits is $d\sin\theta_i$ and that between rays diffracted from successive slits is $d\sin\theta_d$. The path length differences add (rather than subtract as in Figure 3.10) due to the reflective nature of the grating and the way the angles are measured.

However, the maximum energy is not in the zeroth order but in the order corresponding to ordinary (specular) reflection, namely, the order which satisfies $\theta_d - \alpha = \theta_i + \alpha$. For normal incidence, $\theta_i = 0$ and the maixmum energy occurs in the order at the angle $\theta_d = 2\alpha$.

3.6 Assume the slits are located at $\pm d/2, \pm 3d/2, \ldots, \pm(N-1)d/2$ and N is even. Then, the diffracting aperture can be described by

$$f(y) = \frac{1}{N}\sum_{k=1,3,\ldots}^{N-1} \left(\delta(y - kd/2) + \delta(y + kd/2)\right).$$

From (3.11), the amplitude distribution of the diffraction pattern is

$$\begin{aligned} A(\theta) &= \frac{A(0)}{N}\sum_{k=1,3,\ldots}^{N-1} (\exp -2\pi i \sin\theta kd/2\lambda + \exp 2\pi i \sin\theta kd/2\lambda) \\ &= \frac{A(0)}{N}\frac{\sin\frac{2\pi\sin\theta}{\lambda}\frac{Nd}{2}}{\sin\frac{2\pi\sin\theta}{\lambda}\frac{d}{2}} \end{aligned}$$

$A(\theta)$ has maxima when θ satisfies $d\sin\theta = m\lambda$, for some integer m.

As $N \to \infty$, $A(\theta) \to A(0)$, if $d\sin\theta = m\lambda$, for some integer m, and $A(\theta) \to 0$, otherwise. Thus, in the limit of an infinite grating with narrow slits, we get narrow lines of equal amplitude in the diffraction spectrum at the angles corresponding to each grating order.

3.7 The resonant frequencies correspond to the maxima of the transfer function

$$T_{FP}(f) = \frac{1}{1 + \left(\frac{2\sqrt{R}}{1-R}\sin(2\pi f\tau)\right)^2}$$

which occur when $\sin(2\pi f\tau) = 0$ or $2\pi f\tau = k\pi$, where k is an integer. If the resonant frequency f_0 corresponds to k_0, then

$$f_0 = \frac{k_0}{2\tau},$$

and the separation between adjacent resonant frequencies is $\Delta f = \frac{1}{2\tau}$, which is a constant.

3.8 We have waves that make 1 pass, 3 passes, 5 passes, ..., through the cavity before leaving the second mirror. Adding up the contributions by each of these waves, we get the amplitude of the output electric field as

$$E_0 = (1 - A - R)E_i e^{-i\beta l} \sum_{k=0}^{\infty} \left(R\, e^{-i2\beta l}\right)^k.$$

Note that

$$\beta l = \frac{2\pi n l}{\lambda} = f\tau.$$

From the above, the field transfer function is given by

$$\frac{E_0}{E_i} = \frac{(1 - A - R)e^{-i\beta l}}{1 - Re^{-i2\beta l}}.$$

The power transfer function $T_{FP}(\lambda)$ is

$$T_{FP}(\lambda) = \left|\frac{E_0}{E_i}\right|^2 = \frac{(1 - A - R)^2}{1 + R^2 - 2R\cos 2\beta l}.$$

Writing $\cos 2\beta l = 1 - 2\sin^2\beta l$ and simplifying, we get

$$T_{FP}(\lambda) = \frac{\left(1 - \frac{A}{1-R}\right)^2}{1 + \left(\frac{2\sqrt{R}}{1-R}\sin\beta l\right)^2}.$$

3.9 The transfer function of the Fabry-Perot filter is (ignoring absorption)

$$T(f) = \frac{1}{1 + \left(\frac{2\sqrt{R}}{1-R}\sin 2\pi f\tau\right)^2}.$$

In Problem 3.7, we derived the free spectral range (FSR) to be $\frac{1}{2\tau}$. We have $T(f) = \frac{1}{2}$ for f satisfying $\frac{2\sqrt{R}}{1-R}\sin 2\pi f\tau = 1$. If f' is the smallest value of f for which this is satisfied, then the full-width half maximum FWHM $= 2f'$. For R close to 1, that is, $1 - R \ll 1$, f' satisfies

$$\sin 2\pi f'\tau = \frac{1-R}{2\sqrt{R}} \quad \text{or} \quad 2\pi f'\tau \approx \frac{1-R}{2\sqrt{R}}.$$

Hence

$$f' = \frac{1-R}{2\sqrt{R}}\,\frac{1}{2\pi\tau}.$$

Therefore, the finesse F, which is the ratio FSR/FWHM, is given by,

$$\begin{aligned} F &= \frac{FSR}{FWHM} = \left(\frac{1}{2\tau}\right) \Big/ \frac{2(1-R)}{2\pi\tau(2\sqrt{R})} \\ &= \frac{\pi\sqrt{R}}{1-R}. \end{aligned}$$

3.10 The fraction of transmitted energy is

$$t = \frac{\int_{-\infty}^{\infty} T_{FP}(f)\,df}{\int_{-\infty}^{\infty} T_{FP}(0)\,df}.$$

Since the transmission spectrum is periodic, we consider only one FSR to determine the fraction of transmitted energy. We also assume the absorption is negligible so that $A = 0$. Denoting $f\tau = x$, the fraction of transmitted energy is given by

$$\begin{aligned} t &= \int_{-0.5}^{0.5} \frac{dx}{\left(1 + \left(\frac{2\sqrt{R}}{1-R}\sin(2\pi x)\right)^2\right)} \\ &= \frac{1}{\sqrt{1 + \frac{4R}{(1-R)^2}}} \\ &= \frac{1-R}{1+R} \end{aligned}$$

3.11 The FP filter with cavity length l_i, $i = 1, 2$, has a power transfer function,

$$T_i(f) = \frac{1}{\left(1 + \left(\frac{2\sqrt{R}}{1-R}\sin(2\pi f \tau_i)\right)^2\right)}$$

where $\tau_i = l_i n/c$ where n is the refractive index of the cavity, and c is the free space velocity of light. The transfer function of the cascade is

$$T(f) = T_1(f)T_2(f)$$

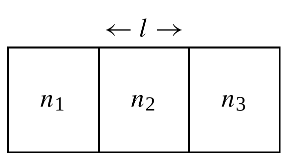
since reflections from the second cavity to the first, and vice versa, are neglected. The maxima of $T(f)$ occur for these values of f which are maxima of both $T_1(f)$ and $T_2(f)$. Thus the FSR of the cascade is

$$\mathrm{FSR} = \mathrm{LCM}(\mathrm{FSR}_1, \mathrm{FSR}_2)$$

where $\mathrm{FSR}_i = c/2nl_i$ is the FSR of the filter with cavity length l_i and LCM denotes the least common multiple. Since $l_1/l_2 = k/m$ and k and m, are relatively prime integers,

$$\mathrm{FSR} = \mathrm{LCM}(\mathrm{FSR}_1, \mathrm{FSR}_2) = k\mathrm{FSR}_1 = m\mathrm{FSR}_2.$$

3.12



Let $\frac{2\pi nl}{\lambda} = x$.

$$Z_{12} = \eta_2 \left(\frac{\eta_3\cos x + i\eta_2\sin x}{\eta_2\cos x + i\eta_3\sin x}\right) = \frac{\eta_0}{n_2}\left(\frac{n_2\cos x + in_3\sin x}{n_3\cos x + in_2\sin x}\right).$$

$$\begin{aligned}
\rho &= \frac{Z_{12}-\eta_1}{Z_{12}+\eta_1} = \frac{(n_2\cos x + in_3\sin x)n_1 - n_2(n_3\cos x + in_2\sin x)}{(n_2\cos x + in_3\sin x)n_1 + n_2(n_3\cos x + in_2\sin x)} \\
&= \frac{i\left(n_1^2-n_2^2\right)\sin x}{2n_1n_n\cos x + i\left(n_1^2+n_2^2\right)\sin x} \qquad\qquad \text{(using } n_1 = n_3\text{)}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
1-|\rho|^2 &= 1-\frac{(n_1^2-n_2^2)^2\sin^2 x}{4n_1^2n_2^2\cos^2 x + (n_1^2+n_2^2)^2\sin^2 x} \\
&= \frac{(2n_1^2)(2n_2^2)\ \sin^2 x + 4n_1^2n_2^2\cos^2 x}{4n_1^2n_2^2\ \cos^2 x + (n_1^2+n_2^2)^2\ \sin^2 x} \\
&= \frac{1}{\cos^2 x + \frac{(n_1^2+n_2^2)^2}{4n_1^2n_2^2}\ \sin^2 x} = \frac{1}{1+\frac{(n_1^2-n_2^2)^2}{4n_1^2n_2^2}\ \sin^2 x}.
\end{aligned}$$

The transfer function of a Fabry-Perot filter with $\sqrt{R} = \frac{n_2-n_1}{n_2+n_1}$ is (using (3.10))

$$\frac{1}{1+\left[\frac{2(n_2-n_1)\ (n_2+n_1)}{(2n_1)(2n_2)}\right]^2\ \sin^2 x} = \frac{1}{1+\frac{(n_1^2-n_2^2)^2}{4n_1^2n_2^2}\ \sin^2 x}$$

which is identical to the expression above.

3.13 We only find the reflectivity at λ_0. For the reflectivity as a function for λ, see M. Born and E. Wolf, *Principles of Optics*, 6th edition, Pergamon Press, Oxford, 1980, Sec. 1.6.5, pp. 66–70.

We assume the surrounding medium is glass with refractive index n_G and intrinsic impedance $\eta_G = \eta_0/n_G$. Denote $\eta_H = \eta_0/n_H$ and $\eta_L = \eta_0/n_L$. Then, by repeated application of (E.2), using $n_H l/\lambda_0 = n_L l/\lambda_0 = 1/4$, we have,

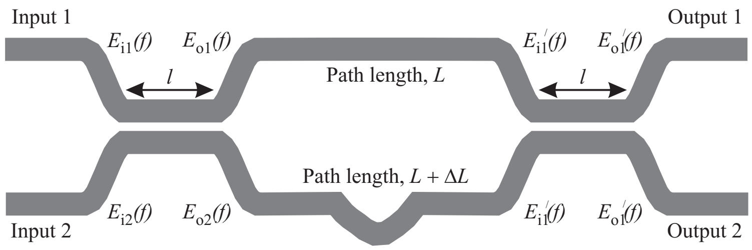
$$\begin{aligned}
Z_{L_kG}(\lambda_0) &= \eta_G, \\
Z_{H_kL_k}(\lambda_0) &= \eta_L^2/\eta_G, \\
Z_{L_{k-1}H_k}(\lambda_0) &= (\eta_H/\eta_L)^2\eta_G, \\
&\vdots \\
Z_{L_1H_2}(\lambda_0) &= (\eta_H/\eta_L)^{2k-2}\eta_G, \\
Z_{H_1L_1}(\lambda_0) &= (\eta_L^{2k}/\eta_H^{2k-2}\eta_G, \\
Z_{GH_1}(\lambda_0) &= (\eta_H/\eta_L)^{2k}\eta_G,
\end{aligned}$$

Thus the reflectivity of the stack at λ_0 is

$$|\rho(\lambda_0)|^2 = \frac{|Z_{GH_1}-\eta_G|^2}{|Z_{GH_1}+\eta_G|^2} = \left(\frac{1-(n_L/n_H)^{2k}}{1+(n_L/n_H)^{2k}}\right)^2.$$

Since $n_L < n_H$, for large k, the reflectivity is almost unity. Thus a stack of alternating high and low refractive index dielectrics which are a quarter-wavelength thick at λ_0, acts as a highly reflective mirror at λ_0.

3.14



Since the directional couplers are 3-dB couplers, from (3.1), with $\kappa l = (2k+1)\pi/4$, for some integer k,

$$\begin{pmatrix} E'_{o1}(f) \\ E'_{o2}(f) \end{pmatrix} = \frac{e^{-i\beta l}}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_{i1}(f) \\ E_{i2}(f) \end{pmatrix}.$$

$$\begin{pmatrix} E'_{i1}(f) \\ E'_{i2}(f) \end{pmatrix} = e^{-i\beta L} \begin{pmatrix} 1 & \\ & e^{-i\beta\Delta L} \end{pmatrix} \begin{pmatrix} E'_{o1}(f) \\ E'_{o2}(f) \end{pmatrix}.$$

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-i\beta l}}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E'_{i1}(f) \\ E'_{i2}(f) \end{pmatrix}.$$

Muliplying the above transfer functions,

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-2i\beta l}}{2} \begin{pmatrix} 1 - e^{-i\beta\Delta L} & i + ie^{-i\beta\Delta L} \\ i + ie^{-i\beta\Delta L} & -1 + e^{-i\beta\Delta L} \end{pmatrix} \begin{pmatrix} E_{i1}(f) \\ E_{i2}(f) \end{pmatrix}.$$

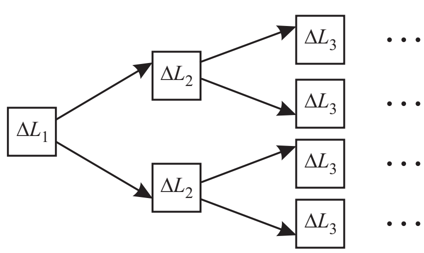
If only one input, say input 1, is active, then $E_{i2}(f) = 0$ and

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-2i\beta l}}{2} \begin{pmatrix} 1 - e^{-i\beta\Delta L} \\ i + ie^{-i\beta\Delta L} \end{pmatrix} E_{i1}(f).$$

The power transfer function is

$$\begin{aligned} \begin{pmatrix} |E_{o1}(f)|^2/|E_{i1}(f)|^2 \\ |E_{o2}(f)|^2/|E_{i1}(f)|^2 \end{pmatrix} &= \frac{1}{4} \begin{pmatrix} (1-\cos\beta\Delta L)^2 + \sin^2\beta\Delta L \\ (1+\cos\beta\Delta L)^2 + \sin^2\beta\Delta L \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 - \cos\beta\Delta L \\ 1 + \cos\beta\Delta L \end{pmatrix} = \begin{pmatrix} \sin^2\beta\Delta L/2 \\ \cos^2\beta\Delta L/2 \end{pmatrix}. \end{aligned}$$

3.15 (a)

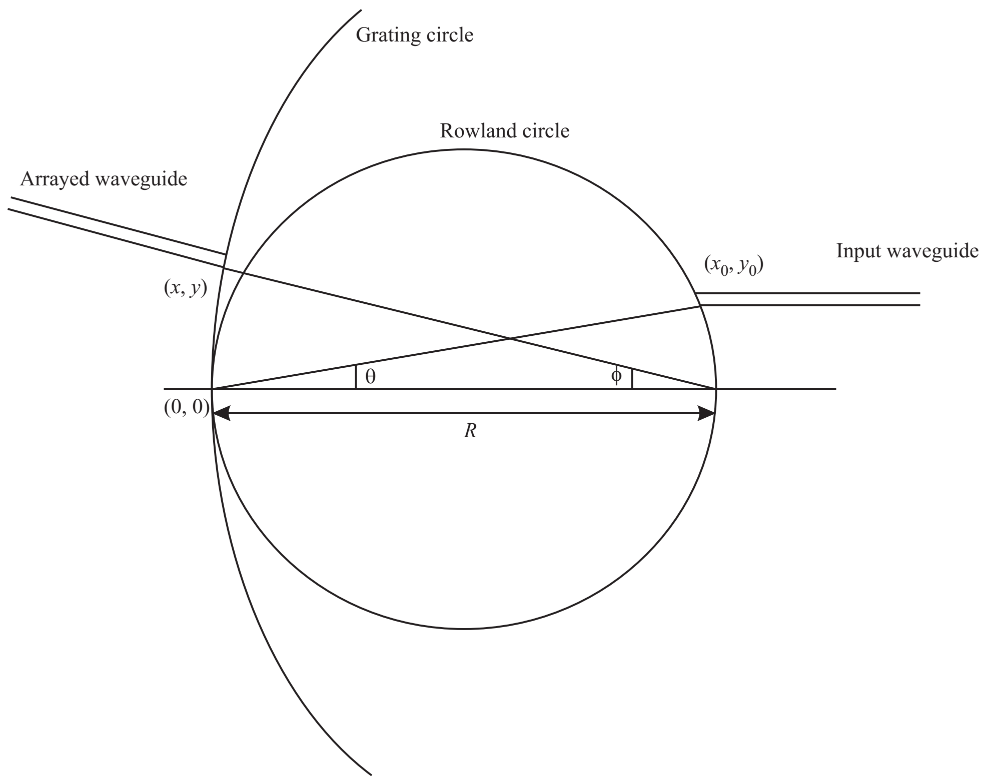


Assume that the frequencies are spaced at $f_0 + i\Delta f$, where $i = 0, 1, \ldots, n-1$. Let $n = 2^k$. Choose n_{eff} and ΔL_1 such that $2\pi f_0 n_{\text{eff}} \Delta L_1/c = k\pi$, for some integer k, and $2\pi n_{\text{eff}} \Delta f \Delta L_1/c = \pi$. Then the top output of the first stage contains the frequencies $f_0 + \Delta f$, $f_0 + 3\Delta f$, $f_0 + 5\Delta f$, …, and the bottom output f_0, $f_0 + 2\Delta f$, $f_0 + 4\Delta f$, …. Choose $\Delta L_2 = \Delta L_1/2, \ldots, \Delta L_k = \Delta L_1/2^{k-1}$.

The choice of ΔL_k only determines the periodicity of the filter. The absolute set of frequencies must be chosen by appropriately varying n_{eff}. The n_{eff} for the filters in each stage must be different in order to accomplish this. For example, in the second stage, the top filter must satisfy $2\pi(f_0 + \Delta f)n_{\text{eff}}\Delta L_1/c = k\pi$, for some integer k, and the bottom filter must satisfy $2\pi f_0 n_{\text{eff}} \Delta L_1/c = k\pi$, for some integer k. However, the n_{eff} differences are slight if $\Delta f \ll f_0$ which is usually the case. Slight changes in n_{eff} can be effected by heating or by applying a voltage (electro-optic effect).

(b) If only one frequency is required, retain only the k MZIs that the desired frequency passes through, in the above construction.

3.16



Let R be the diameter of the Rowland Circle. Then,

$$
\begin{aligned}
x_0 &= R\cos\theta\cos\theta = R\cos^2\theta, \\
y_0 &= R\cos\theta\sin\theta, \\
y &= R\sin\phi \approx R\phi \text{ (for small } \phi), \\
x &= R(1-\cos\phi) = 2R\sin^2(\phi/2) \approx 2R(\phi/2)^2 \text{ (for small } \phi) \\
&\approx y^2/2R.
\end{aligned}
$$

Therefore,

$$\begin{aligned}(x_0 - x)^2 + (y_0 - y)^2 &= (x_0^2 + y_0^2) + x^2 + y^2 - 2xx_0 - 2yy_0 \\ &= R^2\cos^2\theta + y^2 - y^2\cos^2\theta - 2yR\cos\theta\sin\theta \quad (\text{assuming } x^2 \ll y^2) \\ &= (R\cos\theta - y\sin\theta)^2.\end{aligned}$$

(The assumption $x^2 \ll y^2$ amounts to assuming that R is much larger than the length of the arc on which the arrayed waveguides are located.) Therefore, the distance from (x_0, y_0) to $(x, y) = R\cos\theta - y\sin\theta$. If input waveguide i is at an angle θ_i to the central arrayed waveguide and two successive arrayed waveguides are spaced apart vertically by d, the difference in the distances from input waveguide i to these arrayed waveguides is

$$(R\cos\theta_i - y\sin\theta_i) - (R\cos\theta_i - (y + d)\sin\theta_i) = d\sin\theta_i.$$

Using the notation in the book, $d_i^{in} = R\cos\theta_i$ and $\delta_i^{in} = d\sin\theta_i$.

3.17 If

$$n_1\delta_i^{in} + n_2\Delta L + n_1\delta_j^{out} = p\lambda = \frac{pc}{f} = \frac{(p+1)c}{f'},$$

then FSR $= f' - f$ (one period of the transfer function). Therefore,

$$\begin{aligned}\text{FSR} &= \frac{(p+1)c}{n_1\delta_i^{in} + n_2\Delta L + n_1\delta_j^{out}} - \frac{pc}{n_1\delta_i^{in} + n_2\Delta L + n_1\delta_j^{out}} \\ &= \frac{c}{n_1\delta_i^{in} + n_2\Delta L + n_1\delta_j^{out}}.\end{aligned}$$

Using the result of Problem 3.13, $\delta_i^{in} = d\sin\theta_i$ and $\delta_j^{out} = d\sin\theta_j$, where d is the vertical spacing between the arrayed waveguides, θ_i is the angular separation of input waveguide i and the central arrayed waveguide, and θ_j is the angular separation of output waveguide j and the central arrayed waveguide. Therefore,

$$\text{FSR} = \frac{c}{n_1 d\sin\theta_i + n_2\Delta L + n_1 d\sin\theta_j}.$$

If $d \ll \Delta L$,

$$\text{FSR} \approx \frac{c}{n_2\Delta L}.$$

3.18 Consider an $N \times N$ static router of the type shown in Figure 3.22. Using the result of Problem 3.13, from input i, the wavelengths satisfying

$$n_1 d\sin\theta_i + n_2\Delta L + n_1 d\sin\theta_j = p\lambda$$

for some integer p are transferred to output j.

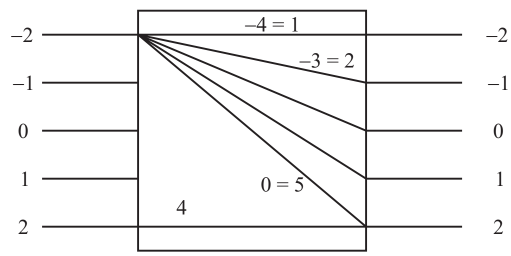
We assume that the angular separation between successive input and output waveguides is $\Delta\theta$. Then we take

$$\theta_i = i\Delta\theta,\ \theta_j = j\Delta\theta,\ i, j = \frac{-(N-1)}{2}, \frac{-(N-3)}{2}, \cdots, -1.0, 1, \cdots \left(\frac{N-1}{2}\right).$$

Here we have assumed that N is odd for simplicity. Thus the inputs and outputs are numbered from $\frac{-(N-1)}{2}$ to $\frac{(N+1)}{2}$.

Let λ_{00} be the wavelength that is transfered from input 0 to output 0. Thus λ_0 satisfies $p\lambda_{00} = n_2\Delta L$. The wavelength λ_{ij} that is transferred from input i to output j satisfies, assuming the θ_i and θ_j are small,

$$n_1 d(i+j)\Delta\theta + n_2\Delta L = p\lambda_{ij}.$$



By renumbering the wavelengths, the static router can be assumed to use wavelength $\lambda_{(i+j) \bmod N}$ to connect input i to output j. The figure above shows the renumbering for $N = 5$. Thus if $(i+j) < 0$, the wavelength used is λ_{i+j+N}. For example, input -2 uses wavelength $-2-2+5 = 1$ to connect to output -2. Thus $\Delta\theta$ must satisfy

$$\begin{aligned} n_1 d(i+j)\Delta\theta + n_2\Delta L &= p\lambda_{ij} \quad \text{and} \\ n_1 d(i+j+N)\Delta\theta + n_2\Delta L &= (p+1)\,\lambda_{ij}. \end{aligned}$$

Therefore,

$$n_1 d N\Delta\theta = \lambda_{ij}.$$

When the FSR is independent of the input and output waveguides, $n_1 d(i+j)\Delta\theta \ll n_2\Delta L$ and $p\lambda_{ij} \approx n_2\Delta L$, for all i, j. If $f_{ij} = c/\lambda_{ij}$ and $f_{ij} + \Delta f = c/\lambda_{i+1,j}$ are adjacent frequencies, using this approximation,

$$\begin{aligned} \Delta f &= \frac{pc}{(n_2\Delta L)^2} n_1 d\Delta\theta \\ &= \frac{pc}{(n_2\Delta L)^2}\frac{\lambda_{ij}}{N} \\ &= \frac{p\lambda_{ij}c}{N(n_2\Delta L)^2} \\ &= \frac{c}{N(n_2\Delta L)} \qquad (\text{using } p\lambda_{ij} \approx n_2\Delta L) \end{aligned}$$

$$= \frac{\text{FSR}}{N}.$$

Thus the N frequencies must be chosen to be equally spaced within an FSR.

3.19 We choose the FSR as 1600 GHz which is the minimum possible value. Since FSR $= c/n_2\Delta L$, assuming $n_2 = 1.5$, $\Delta L = 125\ \mu$m is the path length difference between successive arrayed waveguides.

If the center wavelength is denoted by λ_0, $n_2\Delta L = p\lambda_0$ for some integer p, called the *diffraction order.* Thus $p\lambda_0 = c/\text{FSR} = 187.5\ \mu$m. Choosing $p = 120$, $\lambda_0 = 1.5625\ \mu$m. From Problem 3.15, the spacing between successive frequencies is

$$\Delta f = \frac{pc}{(n_2\Delta L)^2} n_1 d\Delta\theta.$$

Using the values $\Delta f = 100$ GHz, $p = 120$, $\Delta L = 125\ \mu$m, and $n_1 = n_2 = 1.5$, $d\Delta\theta = \Delta f p\lambda_0^2/cn_1 = 0.0651\ \mu$m. Assuming the vertical spacing between successive arrayed waveguides, d, is chosen to be $25\ \mu$m, $\Delta\theta = 2.6 \times 10^{-3}$ radians. If the spacing between successive successive input or output waveguides is $\Delta x = \Delta\theta/R = 25\ \mu$m, we get $R = 9.6$ mm for the diameter of the Rowland circle.

3.20 The transfer function of the AOTF is

$$T(\Delta\lambda) = \frac{\sin^2\left((\pi/2)\sqrt{1 + (2\Delta\lambda/\Delta)^2}\right)}{1 + (2\Delta\lambda/\Delta)^2}.$$

Numerically solving

$$T(\Delta\lambda)/T(0) = 0.5$$

yields

$$\Delta\lambda \approx 0.39\Delta.$$

Hence the FWHM bandwidth of the filter is

$$\approx 0.78\Delta \approx 0.8\lambda_0^2/l\Delta n.$$

3.21 Recall that a polarizer is a 2-input, 2-output device that works as follows. From input 1, the light energy in the TE mode is delivered to output 1, and the light energy in the TM mode is delivered to output 2. Similarly, from input 2, the light energy in the TE mode is delivered to output 2, and the light energy in the TM mode is delivered to output 1. Thus the input polarizer delivers the energy in the TE mode at all wavelengths from input 1, and the TM mode at all wavelengths from input 2, to the upper arm of the AOTF. Similarly, the input polarizer delivers the energy in the TM mode at all wavelengths from input 1, and the TE mode at all wavelengths from input 2, to the lower arm of the AOTF. For the wavelength satisfying the Bragg condition, in the two arms of the polarization-independent AOTF, the light energy undergoes mode conversion, from TE to TM, and vice versa. The output polarizer combines the energy in the TE mode from the upper arm, and the TM mode from the lower arm, and delivers it to output 1. Similarly, it combines the energy in the TM mode from the upper arm, and the TE mode from the lower arm, and delivers it to output 2. Thus all the energy at all the wavelengths, except the one satisfying the Bragg wavelength, are delivered from input 1 to output 1, and input 2 to output 2. Since the energy from the signal at the Bragg wavelength undergoes mode conversion in the two arms of the AOTF, this wavelength is combined by the output polarizer into the "other" output, that is, the signal from input 1 is delivered to output 2, and the signal from input 2 is delivered to output 1. Thus the wavelength satisfying the Bragg condition is exchanged between the two ports.

Multiple wavelengths can be exchanged by launching multiple acoustic waves simultaneously, and the AOTF acts as a 2-input, 2-output wavelength router.

3.22 $\lambda_0 = 1.55\ \mu$m. We take $\Delta n = 0.07$. From the solution of Problem 3.17, for a FWHM of 1 nm,

$$l = \frac{0.8 \times 1.55^2}{10^{-3} \times 0.07}\ \mu\text{m} \approx 27.5\ \text{mm}.$$

3.23

- From the given specifications, we require a free-spectral range (FSR) of ≥ 1600 GHz. For a FP filter, the FSR is given by $1/2\tau$. Thus $\tau \leq 1/3200$ ns. Take $\tau = 1/3200$ ns. We assume the absorption loss $A = 0$ and use (3.10) for the power transfer function of the FP filter. For a 1-dB bandwidth ≥ 2 GHz, $T_{FP}(1) \geq 10^{-0.1} = 0.794$. Solving for R using (3.10), we get, $R \leq 0.992312$, implying the finesse of the filter should be $\leq \pi\sqrt{R}/(1-R) \approx 407$. For a crosstalk suppression of 30 dB from each adjacent channel which is 100 GHz away, we must have, $T_{FP}(100) \leq 10^{-3} = 0.001$. Solving for R using (3.10) yields $R \geq 0.987652$, or a finesse ≥ 253. Thus, to satisfy, the given passband and crosstalk requirements, the reflectivity R must be chosen in the range (0.988, 0.992), for example, 0.99.

  When the center frequencies are allowed to shift by ± 20 GHz from their nominal values, and the filter is *not tunable,* it is impossible to satisfy the crosstalk suppresion requirement of 30 dB. To see this note that we must have
  $$\frac{T_{FP}(80)}{T_{FP}(20)} \leq 10^{-3} = 0.001.$$
  The ratio $T_{FP}(80)/T_{FP}(20)$ decreases monotonically with increasing R; however it is bounded below by a value of 0.0625. To see this approximate $\sin[2\pi f\tau] \approx 2\pi f\tau$, since τ is small, so that the FP transfer function is of the form $(1 + x^2 f^2)^{-1}$ where $x = 2\sqrt{R}/(1-R)2\pi\tau$. Thus
  $$\frac{T_{FP}(80)}{T_{FP}(20)} = \frac{1 + 400x^2}{1 + 6400x^2}$$
  which is a monotonically decreasing function of x^2 bounded below by $400/6400 = 0.0625$.

  If the FP filter is *tunable,* which is the case in some networks, for example, the broadcast-and-select Rainbow network of Chapter 7, then a crosstalk suppression of 30 dB under a center wavelength drift of ± 20 GHz translates to a crosstalk suppression of 30 dB from a channel which is 60 GHz away since the desired channel and the adjacent channel can drift by 20 GHz in opposite directions. Proceeding as above, this yields $R \geq 0.992573$, or a finesse ≥ 421. Since the requirement of a 1-dB passband of 2 GHz yields $R \leq 0.992312$, or a finesse ≤ 407, the two requirements cannot be satisfied simultaneousy. However, a filter with a finesse in the range 410–420, nearly satisfies both requirements.

- For an n-stage Mach-Zehnder interferometer, the transfer function is given by,

$$T_{MZ}(f) = \prod_{k=1}^{n} \cos^2(2^{k-1}\pi f/\text{FSR}).$$

where $\pi f/\text{FSR} = \beta \Delta L/2 = \pi n_{\text{eff}} \Delta L/\lambda$, or $\text{FSR} = c/n_{\text{eff}} \Delta L$. The minimum required FSR is 1600 GHz. If we choose this FSR, and $n = 4$, the nulls in the tranfer function are 100 GHz apart, which is the nominal interchannel spacing. We assume a 4-stage filter so that the crosstalk suppression can be made very large, in the absence of frequency drifts. For an FSR of 1600 GHz, $n_{\text{eff}} \Delta L/c = 1/1600$ ns. For $n_{\text{eff}} = 1.5$, this yields, $\Delta L = 0.2/1600$ m $= 125\ \mu$m for the path length difference of the first stage. The 2nd, 3rd and 4th stages have path length differences of 250, 500 and 1000 μm, respectively.

  At 1 GHz away from the center frequnecy, the transfer function is $T_{MZ}(1) = 0.999672$ so that the requirement of a 1-dB bandwidth larger than 2 GHz is easily satisfied. The null at 100 GHz is very sharp since $T_{MZ}(90) = 0.012$ and $T_{MZ}(110) = 0.008$. This already suggests that the filter design may not be possible in the presence of a frequency drift of ± 20 GHz which is indeed the case. To see this note that increasing the FSR will make the crosstalk suppression worse since, relative to the FSR, the interchannel spacing will be smaller. So we use the minimum FSR of 1600 GHz. For this FSR, if we use 6 stages, the nulls occur 25 GHz apart. We cannot use more stages since the center frequency of a channel may then drift into a null of the transfer function. For 6 stages or fewer, numerical calculations of $T_{MZ}(20)/T_{MZ}(f')$ where $f' \in [80, 120]$ GHz show that a crosstalk suppression of 30 dB is not achievable.

  If the filter is tunable, a crosstalk suppression of over 30 dB can be achieved by using an 8-stage filter with an FSR of 1600 GHz. The worst-case crosstalk of 30.3 dB occurs when the adjacent channel is 65.565 GHz away from the desired channel. The 8-stage filter also satisfies the 2 GHz 1-dB bandwidth requirement.

- In the case of the AOTF, the transfer function nulls are not spaced equally apart in *frequency*, as in the case of the multistage MZI. So we cannot design the filter with a low crosstalk suppression (in the absence of frequency drifts) by making the transfer function nulls coincide with the channel positions. (Far away from the main lobe, the nulls are approximately equally spaced in *wavelength*.) Since the first side lobe is less than 10 dB below the main lobe, this suggests that for a filter meeting the specified requirements, the adjacent channel must occur after several transfer function nulls. This is indeed the case and a filter can be designed (in the absence of frequency drifts) as follows.

  We note that

$$T_{AOTF}(\lambda) = \frac{\sin^2\left((\pi/2)\sqrt{1+(2\Delta\lambda/\Delta)^2}\right)}{1+(2\Delta\lambda/\Delta)^2}. \le 1/(1+(2\Delta\lambda/\Delta)^2) = T'(\lambda).$$

In the 1.55 μm band, a spacing of 100 GHz $\approx$ 0.8 nm. Solving $T'(0.8) = 10^{-3}$ yields, $0.8/\Delta = 15.8035$ or $\Delta = \lambda_0^2/l\Delta n = 0.8/15.8$. Assuming $\Delta n = 0.07$ and $\lambda_0 = 1.55\ \mu$m yields $l \approx 68$ cm. For an integrated optics AOTF, this is a highly impractical value of l, which again illustrates the poor crosstalk suppression capabilities of the AOTF compared to other structures. However this (impractical) filter does satisfy the 2 GHz 1-dB bandwidth requirement.

  In the presence of frequency drifts, similar problems arise as in the FP and MZI structures, and the filter design is impossible. In the tunable filter case, the crosstalk suppression of 30 dB must now occur for a worst-case spacing of only 60 GHz which makes the required value of l even larger at 113 cm! In this case the 1-dB bandwidth is slightly less than the required value of 2 GHz; the transfer function is 1.25 dB down at 1 GHz on either side of the center frequency.

Note that none of these filters are capable of handling a variation of 20 GHz in the channel positions. In practical applications, the passband shape is engineered to have a flatter top and sharper skirts to meet this requirement.

3.24 (a) Structure of Figure 3.14(b):
The loss for a dropped channel = 1 dB (first pass) + 1 dB (second pass) = 2 dB.
The loss for an added channel = 13 dB (input coupling loss of 5% tap).
The loss for a passed-through channel = 1 dB (first pass) + 0.5 dB (grating loss) + 0.2 dB (coupling loss) = 1.7 dB.
Power of a passed-through channel = −15 dBm − 1.7 dB = −16.7 dBm.
For the added channel to have the same power, it must be transmitted at = −16.7 dBm + 13 dB = −3.7 dBm.

(b) Structure of Figure 3.14(b) cascaded:
Dropped channel worst-case loss = 1.7 dB × 3 (three passes-through) +2 dB = 7.1 dB.
Dropped channel best-case loss = 1.7 dB × 0 (no passes-through) +2 dB = 2 dB.
Added channel worst-case loss = 13 dB + 1.7 dB × 3 = 18.1 dB.
Added channel best-case loss = 13 dB + 1.7 dB × 0 = 13 dB.
Pass-through channel worst-case and best-case loss = 1.7 dB × 4 = 6.8 dB.

(c) Structure of Figure 3.82:
For this structure, the best-case and worst-case losses are the same.
Dropped channel worst-case loss = 2 dB + 6 dB (splitting loss) +1 dB (filter) = 9 dB.
Added channel worst-case loss = 6 dB (combining loss) +10 dB (input coupling loss) = 16 dB.
Passed-through channel worst-case loss = 1 dB (circulator pass) +2 dB (grating pass) +0.5 dB (output coupling loss) = 4.5 dB.

Comparing with the results of (b), we see that the structure of Figure 3.60 has a lower worst-case loss for the added and passed-through channels. Moreover the loss is uniform.

(d) The costs of the two structures are compared in the following table.

	Figure 3.14(b) cascaded	Figure 3.60
Fiber grating	\$2,000	\$2,000
Circulators	\$12,000	\$3,000
Filters	–	\$4,000
Splitters/combiners	–	\$200
Couplers	\$400	\$100
Total	\$14,400	\$9,300

Thus, from the cost viewpoint also, the structure of Figure 3.60 is better.

3.25 Conduction band electrons in a photodetector do not absorb incident photons since there are no higher energy levels or band to which they can be excited.

3.26 (a) See Figure 3.34. To minimize ASE, pump in the forward direction. To prevent back reflections at the input add an isolator at the amplifier input.

(b) See Figure 3.35.

(c) 1532 nm corresponds to 195.82 THz. 1550 nm corresponds to 193.55 THz. The total bandwidth available is therefore 2200 GHz. With 100 GHz spacing we can have 22 channels within this band.

(d) The required energies are given by hf, where h is Planck's constant and f is the frequency. Using $h = 6.63 \times 10^{-34}$ J/Hz, the energy range required is 1.283×10^{-19} J to 1.298×10^{-19} J.

(e) This would be a two stage EDFA shown in Figure 3.37 with the loss element replaced by the ADM.

(f) See Figure 3.82, with only two fiber Bragg gratings and a 2×1 combiner and splitter. Note that there are multiple alternatives. Another alternative is to cascade two individual add/drop units, one for each wavelength, each with a circulator and combiner. Another is to cascade two individual drops first and combine the adds later.

(g) >From Section 3.3.3., the period of the grating is given by $\Lambda = \lambda_0/2n_{eff}$. Using $n_{eff} = 1.5$, we get the periods of the gratings corresponding to 1532 and 1532.8 nm being 510.667 nm and 510.933 nm.

3.27 If a switch has a crosstalk suppression of 50 dB, it means that the input power from each other input is 10^{-5} of the input power from the desired input. In a 4×4 switch, we have three other (unwanted) inputs. In the worst-case, we get each of them at the desired output with a relative attenuation of 10^{-5}. Thus the crosstalk suppression is $-10 \log_{10} 3 \times 10^{-5} \approx 45$ dB.

If the overall crosstalk suppression should be 40 dB, we need 45 dB crosstalk suppression in each 2×2 switch.

3.28 First, the resulting filter will have a peak whenever $mf_1 = nf_2$, where m and n are integers. Since $lcm(f_1, f_2)$ is a multiple of both f_1 and f_2, the resulting filter has a peak at $q lcm(f_1, f_2)$, where q is an integer. We now prove that there cannot be a peak between $q lcm(f_1, f_2)$ and $(q+1) lcm(f_1, f_2)$.

Pick the two integers M and N such that $Mf_1 = Nf_2 = q lcm(f_1, f_2)$. The next peak will then occur at the smallest values of two integers i and j such that

$$(M + i) f_1 = (N + j) f_2,$$

or

$$i f_1 = j f_2.$$

By definition the smallest values of i and j for which this is satisfied is $lcm(f_1, f_2)$. Therefore the resulting filter is periodic with period $lcm(f_1, f_2)$.

3.29 Say we have to establish a connection from an idle input of a first stage switch X to an idle output of the third stage switch Y. To do so we have to have a fanout such that we can always find a middle stage switch. Note that since at most $m - 1$ outputs of the switch X can be busy. Likewise, at most $m - 1$ inputs of the switch Y can be busy. To establish a connection we need to find a middle stage switch to which one of these free ports in switch X and switch Y are connected. In the worst case, the busy outputs of switch X are connected to $m - 1$ separate mid-stage switches and the busy outputs of switch Y are connected to $m - 1$ other separate mid-stage switches. To find a free mid-stage switch, there we must have the number of mid-stage switches, $p \geq (m - 1) + (m - 1) = 2m - 1$.

4 chapter

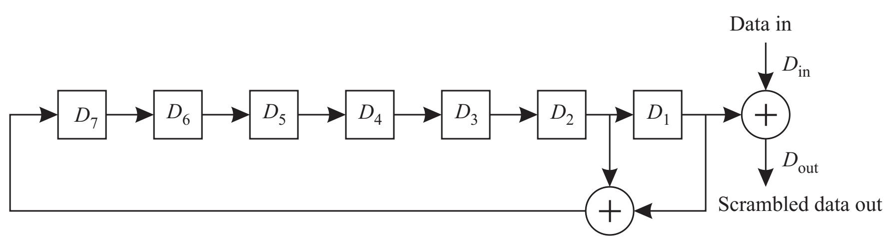
Modulation and Demodulation

4.1 (a) 1111101111101001000000.

(b) 01111101011111100011 is decoded as 011111 1011111 0011 .

The algorithm used by the decoder is to omit the zero following a sequence of five 1s.

4.2 (a)



The bits are assumed to be labelled as in the figure above. The operation of the shift register is shown in the table below.

D_{in}	$D_{out} = D_{in} + D_1$	D_7	D_6	D_5	D_4	D_3	D_2	D_1
–	–	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1
1	0	0	0	0	1	1	1	1
1	0	0	0	0	0	1	1	1
1	0	0	0	0	0	0	1	1
1	0	0	0	0	0	0	0	1
1	0	1	0	0	0	0	0	0
⋮	⋮				⋮			

The scrambled output is 00000001111000000001.

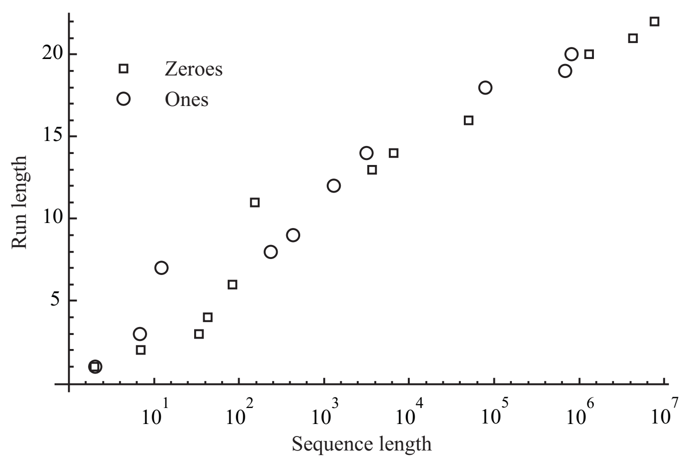
(b) The C program for solving this problem is given below.

```
#include <stdio.h>
#include <stdlib.h>
#define MAXBITS 10000000
main()
{
   int D[8], Din, Dout;
   int i, j, temp, prevbit, seq[2], maxseq[2];

   /* Initialize random number generator */
   srandom(1);

   /* Intialize shift register contents */
   for(j = 1; j <= 7; ++j) D[j] = 1;
   seq[0] = seq[1] = 0;
   maxseq[0] = maxseq[1] = 0;

   printf(" k   Sequence    Max run\n");
   printf("       length     of k's\n");

   prevbit = 2;
   for(i = 1; i <= MAXBITS; ++i)
   {
      Din = random()&01; /* Din is a random bit */
      Dout = Din^D[1];
      temp = D[1]^D[2];
      for(j = 1; j < 7; ++j) D[j] = D[j+1];
      D[7] = temp; /* Shifting of bits */

      if(Dout == prevbit) ++seq[Dout]; /* run continues */
      else /* run has ended */
      {
         seq[Dout] = 1;
         if(seq[Dout^1] > maxseq[Dout^1])
         {
            /* new maximum run length found */
            maxseq[Dout^1] = seq[Dout^1];
            printf(" %1d %10d %10d\n",
               Dout^1,i,maxseq[Dout^1]);
         }
      }
      prevbit = Dout;
   }
}
```

For one sample run, the observed output was as follows.

k	Sequence length	Max run of k's
0	2	1
1	3	1
0	7	2
1	10	3
1	18	7
0	34	3
0	43	4
0	83	6
0	153	11
1	348	8
1	635	9
1	1940	12
0	3682	13
1	4664	14
0	6631	14
0	49942	16
1	117432	18
1	1008625	19
1	1200797	20
0	1301644	20
0	4299910	21
0	7597862	22

This is plotted in the figure below.



4.3

$$d(nT) = 10101011010111100001$$

$$x(nT) = 11001101100101000001$$

$$\begin{aligned} x(nT-T) &= 011001101100101000001 \\ y(nT) &= x(nT)+x(nT-T) \\ &= 121012112101111100001 \end{aligned}$$

To get $x(nT)$ from $d(nT)$ we assume $d(nT)$ is preceded by 0's. Note that $y(nT) \bmod 2 = d(nT)$ as expected.

Formally, the differential encoding $x(nT)$ of $d(nT)$ is obtained using

$$x(nT) = [x(nT-T)+d(nT)] \bmod 2 = \sum_{i=0}^{\infty} d(nT-iT) \bmod 2.$$

Thus, $x(nT)$ is the running sum, modulo 2, of $d(nT)$.

Therefore,

$$y(nT) = x(nT)+x(nT-T) = \sum_{i=0}^{\infty} d(nT-iT) \bmod 2 + \sum_{i=1}^{\infty} d(nT-iT) \bmod 2.$$

Thus, $y(nT) = 1$, if $d(nT) = 1$, and $y(nT) = 2x(nT-T) = 2\left(\sum_{i=1}^{\infty} d(nT-iT) \bmod 2\right)$, if $d(nT) = 0$.

4.4

$$\mathrm{SNR} = \frac{\overline{I}^2}{\sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2} = \frac{(G_m \mathcal{R} P)^2}{2eG_m^2\, F_A(G_m)\mathcal{R}PB_e + \frac{4k_BT}{R_L}F_nB_e}.$$

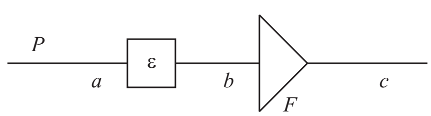
To optimize the SNR, we set

$$\begin{aligned} &\frac{\partial \mathrm{SNR}}{\partial G_m} = 0, \\ \text{or } &\left(\frac{\partial \overline{I}^2}{\partial G_m}\right)(\sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2) - \left(\frac{\partial \sigma_{\text{shot}}^2}{\partial G_m}\right)\overline{I}^2 = 0 \\ \text{or } &\left(2eG_m^2 F_A(G_m)\mathcal{R}PB_e + \sigma_{\text{thermal}}^2\right) - G_m e(x+2)G_m^{x+1}\,\mathcal{R}PB_e = 0, \end{aligned}$$

where we have used $F_A(G_m) = G_m^x$. Solving this equation yields

$$G_m = \left(\frac{\sigma_{\text{thermal}}^2}{e\mathcal{R}PB_e x}\right)^{\frac{1}{x+2}} = \left(\frac{4k_BTF_n}{eR_L\mathcal{R}Px}\right)^{\frac{1}{x+2}}.$$

4.5 (a)



We have

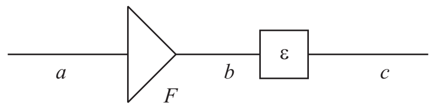
$$F = \frac{\mathrm{SNR}_b}{\mathrm{SNR}_c}.$$

The signal-to-noise ratios at points a and b, respectively, are

$$\mathrm{SNR}_a = \frac{(\mathcal{R}P)^2}{2\mathcal{R}ePB_e} \text{ and } \mathrm{SNR}_b = \frac{(\mathcal{R}P(1-\epsilon))^2}{2\mathcal{R}eP(1-\epsilon)B_e}.$$

Therefore
$$\frac{\mathrm{SNR}_a}{\mathrm{SNR}_b} = \frac{1}{1-\epsilon}.$$
The overall noise figure is given by
$$\frac{\mathrm{SNR}_a}{\mathrm{SNR}_c} = \frac{F}{1-\epsilon}.$$

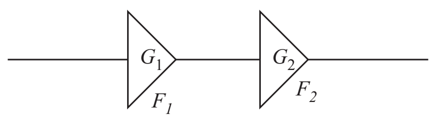
(b)



We have
$$F = \frac{\mathrm{SNR}_a}{\mathrm{SNR}_b}.$$
$$\mathrm{SNR}_b = \frac{\mathcal{R}P}{4\mathcal{R}^2 P n_{sp}\, hf_c\, B_e}$$
and
$$\mathrm{SNR}_c = \frac{\mathcal{R}P(1-\epsilon)}{4\mathcal{R}^2 P(1-\epsilon) n_{sp}\, hf_c\, B_e} = \mathrm{SNR}_b.$$
Therefore the overall noise figure is F.

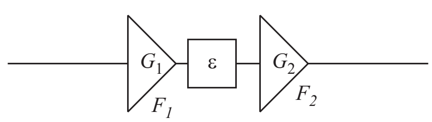
Note that this is true only if the signal-spontaneous noise power at c is much larger than the receiver thermal noise power, which will be the case for power levels that are several dB higher than the receiver sensitivity.

(c)



Note that $F_1 \approx 2n_{sp1}$ and $F_2 \approx 2n_{sp2}$. Therefore, at the output, the noise power is given by
$$F_1 h\nu(G_1-1)G_2 B_0 + F_2 h\nu(G_2-1)B_0 = [F_1(G_1-1)G_2 + F_2(G_2-1)]\, h\nu B_0.$$
Consider an equivalent amplifier with gain G_1G_2. Its noise power is
$$Fh\nu(G_1G_2-1)B_0.$$
Comparing this with the equation above, we get
$$F = \frac{F_1(G_1-1)G_2 + F_2(G_2-1)}{G_1G_2-1}.$$
Assuming $G_1, G_2 \gg 1$, we have
$$F = F_1 + \frac{F_2}{G_1}.$$

(d)



This is similar to 4.4(c). The noise power at the output is given by
$$[F_1(G_1-1)(1-\epsilon)G_2 + F_2(G_2-1)]h\nu B_0.$$
The equivalent amplifier has gain $G_1G_2(1-\epsilon)$ and its noise power is
$$Fh\nu(G_1G_2(1-\epsilon)-1)B_0.$$

Assuming $G_1(1-\epsilon) \gg 1$ and $G_2 \gg 1$, we get

$$F = F_1 + \frac{F_2}{G_1(1-\epsilon)}.$$

4.6 We have

$$\text{BER} = \frac{1}{2}P[1|0] + \frac{1}{2}[1|0] = \frac{1}{2}Q\left(\frac{I_1 - I_{th}}{\sigma_1}\right) + \frac{1}{2}Q\left(\frac{I_{th} - I_0}{\sigma_0}\right).$$

Using

$$I_{th} = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$$

from (4.12), we get

$$I_1 - I_{th} = \frac{\sigma_1(I_1 - I_0)}{\sigma_0 + \sigma_1}$$

and

$$I_{th} - I_0 = \frac{\sigma_0(I_1 - I_0)}{\sigma_0 + \sigma_1}.$$

Therefore, we have

$$\text{BER} = \frac{1}{2}Q\left(\frac{I_1 - I_0}{\sigma_0 + \sigma_1}\right) + \frac{1}{2}Q\left(\frac{I_1 - I_0}{\sigma_0 + \sigma_1}\right) = Q\left(\frac{I_1 - I_0}{\sigma_0 + \sigma_1}\right).$$

4.7

$$\text{BER} = \frac{1}{2}Q\left(\frac{m_1 - T_d}{\sigma_1}\right) + \frac{1}{2}Q\left(\frac{T_d - m_0}{\sigma_0}\right),$$

From this expression, for large $|T_d|$, BER $\rightarrow 1/2$, so that an optimum T_d that minimizes the BER exists. Setting $\partial\text{BER}/\partial T_d = 0$, we get,

$$\frac{1}{\sigma_1}e^{-(m_1-T_d)^2/2\sigma_1^2} = \frac{1}{\sigma_0}e^{-(T_d-m_0)^2/2\sigma_0^2}$$

or

$$\frac{(T_d - m_0)^2}{2\sigma_0^2} - \frac{(m_1 - T_d)^2}{2\sigma_1^2} = \ln \sigma_1/\sigma_0$$

which can be written as

$$(\sigma_1^2 - \sigma_0^2)T_d^2 + 2(m_1\sigma_0^2 - m_0\sigma_1^2)T_d + m_0^2\sigma_1^2 - m_1^2\sigma_0^2 - 2\sigma_0^2\sigma_1^2 \ln \sigma_1/\sigma_0 = 0.$$

Solving this quadratic equation for T_d, we get (4.21).

4.8 From (4.15), the receiver sensitivity

$$\overline{P}_{rec} = \frac{(\sigma_0 + \sigma_1)}{2G_m\mathcal{R}}.$$

Neglecting shot noise, we have $\sigma_0^2 = \sigma_1^2 = \sigma_{\text{thermal}}^2$ and $G_m = 1$ for a *pin* receiver. Therefore,

$$\bar{P}_{\text{sens}} = \frac{\sigma_{\text{thermal}}\gamma}{\mathcal{R}}.$$

The power per 1 bit $P_1 = 2\overline{P}_{\text{rec}}$. For an error rate of 10^{-12}, $\gamma = 7$. Using $\sigma_{\text{thermal}} = 1.656 \times 10^{-22}\, B$, we get

$$P_1 = \frac{2 \times \sqrt{1.656 \times 10^{-22} B} \times 7}{1.25} = 1.44 \times 10^{-10}\sqrt{B}\text{ W}$$

At $B = 100$ Mb/s, $P_1 = 1.44\,\mu$W and $M = 1.12 \times 10^5$ photons per 1 bit. At $B = 1$ Gb/s, $P_1 = 4.56\,\mu$W and $M = 35.5 \times 10^3$ photons per 1 bit.

4.9 (a) $$\bar{P}_{\text{sens}} = \frac{(\sigma_0 + \sigma_1)\gamma}{2G_m\mathcal{R}} \text{ or } \left(\bar{P}_{\text{sens}} - \frac{\sigma_0\gamma}{2G_m\mathcal{R}}\right)^2 = \left(\frac{\sigma_1\gamma}{2G_m\mathcal{R}}\right)^2.$$

Using $\sigma_{\text{shot}}^2 = 4eG_m^2 F_A(G_m)\mathcal{R}\,\bar{P}_{\text{sens}}B_e$, $\sigma_0^2 = \sigma_{\text{thermal}}^2$ and $\sigma_1^2 = \sigma_{\text{thermal}}^2 + \sigma_{\text{shot}}^2$ in the equation above, we get

$$\bar{P}_{\text{sens}} = \frac{\gamma}{\mathcal{R}}\left(\frac{\sigma_{\text{thermal}}}{G_m} + \gamma e F_A(G_m)B_e\right).$$

(b) To obtain the optimum value of G_m, we set

$$\frac{\partial \bar{P}_{\text{sens}}}{\partial G_m} = 0 \Rightarrow \frac{-\sigma_{\text{thermal}}}{G_m^2} + \gamma e B_e \frac{\partial F_A}{\partial G_m} = 0.$$

Note that

$$F_A(G_m) = k_A G_m + (1 - k_A)(2 - 1/G_m).$$

So we have

$$\frac{\partial F_A}{\partial G_m} = k_A + \frac{(1 - k_A)}{G_m^2}.$$

Substituting this in the equation above, we obtain

$$\gamma e B_e \left[k_A + \frac{(1-k_A)}{G_m^2}\right] - \frac{\sigma_{\text{thermal}}}{G_m^2} = 0$$

$$\text{or } (\gamma e B_e k_A)G_m^2 = \sigma_{\text{thermal}} - (1 - k_A)\gamma e B_e$$

$$\text{or } G_m = \sqrt{\frac{\sigma_{\text{thermal}}}{\gamma e B_e k_A} - \frac{1-k_A}{k_A}}.$$

(c) From the solution to (b), we have

$$\frac{\sigma_{\text{thermal}}}{G_m^{\text{opt}}} = \gamma e B_e \left[k_A + \frac{1 - k_A}{(G_m^{\text{opt}})^2}\right] G_m^{\text{opt}}.$$

Therefore,

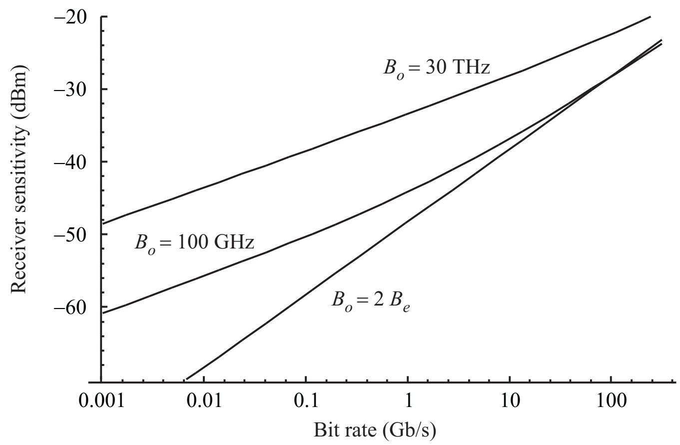
$$\begin{aligned}
\bar{P}_{\text{sens}} &= \frac{\gamma}{\mathcal{R}}\left[\gamma e B_e G_m^{\text{opt}} k_A + \gamma e B_e k_A G_m^{opt} + 2(1 - k_A)\gamma e B_e\right] \\
&= \frac{\gamma}{\mathcal{R}}\left[2\gamma e B_e G_m^{opt} k_A + 2(1 - k_A)\gamma e B_e\right] \\
&= \frac{2\gamma^2 e}{\mathcal{R}} B_e \left[k_A G_m^{opt} + 1 - k_A\right].
\end{aligned}$$

4.10 We assume that $I_1 = \mathcal{R}GP$ (using (4.6)), $I_0 = 0$, $\sigma_1 = \sigma_{\text{sig-spont}}$, and $\sigma_0 \ll \sigma_1$. With these assumptions, (4.14) reduces to,

$$\text{BER} = Q\left(\frac{I_1 - I_0}{\sigma_0 + \sigma_1}\right) = Q\left(\frac{\mathcal{R}GP}{\sqrt{4\mathcal{R}^2 G P P_n (G - 1)B_e}}\right)$$

where we have used (4.9) for $\sigma_1 = \sigma_{\text{sig-spont}}$. This simplifies to (4.18).

4.11



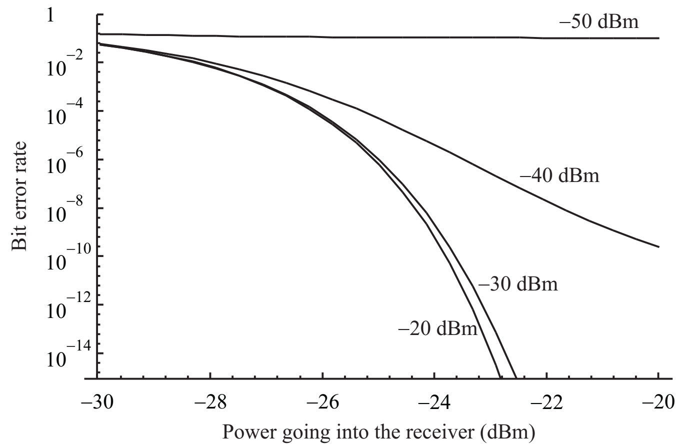
The sensitivity is plotted in the figure above. We assume the amplifier gain is reasonably large so that the thermal and shot noise terms can be neglected. So we consider only the signal-spontaneous and spontaneous-spontaneous terms given by (4.9) and (4.10), respectively. The receiver sensitivity is obtained by solving for $\bar{P}_{\text{sens}}$ in (4.15), using both the terms for σ_1 and only the spontaneous-spontaneous term for σ_0. The resulting expression which is plotted in the figure above is

$$\bar{P}_{\text{sens}} = \gamma \frac{e}{\mathcal{R}} F_n \frac{B}{2} \left(\gamma + \sqrt{\frac{2B_o - B/2}{B}} \right)$$

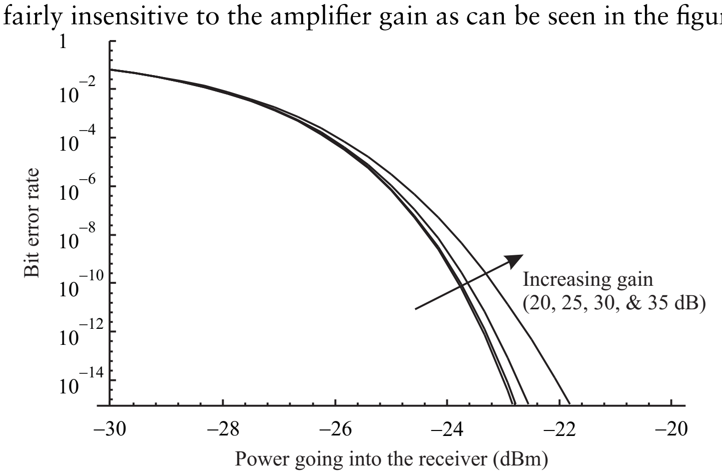
where we have used the condition $B_e = B/2$.

4.12 We assume the optical amplifier has a gain $G = 30$ dB; however the results are fairly insensitive to the gain as we will see later.

Denote the received power at the input of the optical amplifier by P and the loss introduced by the attenuator by L. Both the signal and the spontaneous emission from the optical amplifier are attenuated by L. Thus the noise variances in (4.7)–(4.10) are modified with GP replaced by GPL and $P_n(G-1)$ replaced by $P_n(G-1)L$. We calculate the BER using (4.14) where we set $P = 2\bar{P}_{\text{sens}}$ for a '1' bit, and $P = 0$ for a '0' bit. We plot the BER versus the the signal power going into the receiver namely, GPL, when L is varied, for four different values of P namely, −20, −30, −40, and −50 dBm.



When the power into the amplifier is high ($P = -20$ and -30 dBm), the attenuator needs to be set to a high loss value to measure BERs in the range of 10^{-12} to 10^{-3}. Due to the high attenuation, the receiver is essentially thermal noise limited in this case and this is seen in the curves, where the BER drops significantly as the power into the receiver is increased. For lower received signal powers, such as $P = -40$ dBm, the attenuator is set to a low to moderate loss value, and in this case, the receiver performance is dominated by the signal-spontaneous noise. For this case, increasing the power into the receiver by varying the attenuator setting doesn't have as much of an impact on the BER as can be seen by the levelling off of the BER curve. The receiver performance is fairly insensitive to the amplifier gain as can be seen in the figure below.



Here we plot the BER versus the signal power going into the receiver for $P = -30$ dBm for $G = 20$, 25, 30, and 35 dB. For G in the 20–30 dB range, the curves are very close to each other. For $G = 35$ dB, the signal-spontaneous noise begins to dominate and the BER increases for the same signal power into the receiver.

4.13 The OSNR is defined as the ratio of average signal power to the total noise power in both polarization modes. Assuming that P is the average power, we can write

$$\mathrm{OSNR} = \frac{GP}{P_{\mathrm{ASE}}}$$

where $P_{\text{ASE}} = 2P_n(G-1)B_o$ is the total noise power in both polarization modes.

For a 1 bit, we can now rewrite (4.9) as

$$\sigma^2_{\text{sig-spont}} = 4\mathcal{R}^2 G(2P)P_n(G-1)B_e = 4\mathcal{R}^2 GPP_{\text{ASE}}\frac{B_e}{B_o},$$

and for both a 0 and 1 bit, (4.10) becomes

$$\sigma^2_{\text{spont-spont}} = 2\mathcal{R}^2[P_n(G-1)]^2(2B_o - B_e)B_e \approx \mathcal{R}^2 P^2_{\text{ASE}}\frac{B_e}{B_o}.$$

Here we have assumed that $2B_o \gg B_e$, which is the case in most practical systems. Therefore we have

$$\begin{aligned}
\gamma &= \frac{\mathcal{R}2GP}{\sqrt{4\mathcal{R}^2 GPP_{\text{ASE}}\frac{B_e}{B_o} + \mathcal{R}^2 P^2_{\text{ASE}}\frac{B_e}{B_o}} + \sqrt{\mathcal{R}^2 P^2_{\text{ASE}}\frac{B_e}{B_o}}} \\
&= \frac{2\mathcal{R}P_{\text{ASE}}\text{OSNR}}{\sqrt{4\mathcal{R}^2\text{OSNR}P^2_{\text{ASE}}\frac{B_e}{B_o} + \mathcal{R}^2 P^2_{\text{ASE}}\frac{B_e}{B_o}} + \sqrt{\mathcal{R}^2 P^2_{\text{ASE}}\frac{B_e}{B_o}}} \\
&= \frac{2\text{OSNR}\sqrt{\frac{B_o}{B_e}}}{1+\sqrt{1+4\text{OSNR}}}.
\end{aligned}$$

For large signal-to-noise ratios ($4\text{OSNR} \gg 1$), this can be expressed as

$$\gamma = \sqrt{\text{OSNR}\frac{B_o}{B_e}}.$$

4.14 For a PSK homodyne receiver,

$$\begin{aligned}
I_1 &= \mathcal{R}(P + P_{LO} + 2\sqrt{PP_{LO}}), \\
I_0 &= \mathcal{R}(P + P_{LO} - 2\sqrt{PP_{LO}}).
\end{aligned}$$

We have

$$\begin{aligned}
\sigma_1^2 &= 2eI_1B_e \approx 2e\mathcal{R}B_eP_{LO}, \\
\sigma_0^2 &= 2eI_1B_e \approx 2e\mathcal{R}B_eP_{LO}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{BER} &= Q\left(\frac{4\mathcal{R}\sqrt{PP_{LO}}}{2\sqrt{2e\mathcal{R}B_eP_{LO}}}\right) \\
&= Q\left(2\sqrt{\frac{\mathcal{R}P}{2eB_e}}\right).
\end{aligned}$$

Assuming $B_e = B/2$, we get

$$\text{BER} = Q\left(2\sqrt{\frac{\mathcal{R}P}{eB}}\right) = Q(2\sqrt{M}),$$

where M is the number of photons per 1 bit. For a BER of 10^9, we want $2\sqrt{M} = 6$ or $M = 9$ photons / 1 bit.

4.15 Let the signal field at the input be

$$E_s = \sqrt{2aP}\cos(2\pi f_c t)$$

and the local oscillator field at the input be

$$E_{LO} = \sqrt{2P_{LO}}\cos(2\pi f_c t).$$

If we use a $\frac{\pi}{2}$ phase shift at the second input and output of the coupler, its scattering matrix becomes

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Therefore, the fields going into the two detectors are

$$\begin{aligned} E_1 &= (E_s + E_{LO})/\sqrt{2}, \quad \text{and} \\ E_2 &= (E_s - E_{LO})/\sqrt{2}. \end{aligned}$$

The equivalent powers are

$$\begin{aligned} P_1 &= \frac{1}{2}\left(E_s^2 + E_{LO}^2 + 2E_s E_{LO}\right) \\ &= \frac{1}{2}\left(aP + P_{LO} + 2\sqrt{aPP_{LO}}\right), \quad \text{and} \\ P_2 &= \frac{1}{2}\left(aP + P_{LO} - 2\sqrt{aPP_{LO}}\right). \end{aligned}$$

The average difference current is given by

$$i = \mathcal{R}(P_1 - P_2) = \frac{1}{2}\mathcal{R}4\sqrt{aPP_{LO}} = 2\mathcal{R}\sqrt{aPP_{LO}}.$$

On the other hand, the average noise power is the sum of the noise powers in the two arms and is given by

$$\begin{aligned} \sigma^2 &= 2e\mathcal{R}P_1 B_e \times 2 \\ &= 2e\mathcal{R}B_e P_{LO}. \end{aligned}$$

The BER is given by (for OOK)

$$Q\left(\frac{2\mathcal{R}\sqrt{PP_{LO}}}{2\sqrt{2e\mathcal{R}B_e P_{LO}}}\right) = Q\left(\sqrt{\frac{\mathcal{R}P}{2eB_e}}\right),$$

which is the same as the expression derived in Section 4.4.7.

4.16 If the bits to be transmitted are 010111010111101111001110, the parity check bits are 11101000 so that the transmitted sequence is 01011101011110111100111011101000.

If the received bits are 010111010111101111001110, the received parity bits are the last 8 bits, namely, 11001110. Computing the BIP-8 on the first 24 bits yields 11101000. The number of mismatches between the received parity checks and the computed parity checks is 3 and hence, we conclude, that 3 bit-errors occurred.

4.17 The receiver makes an error provided that either all three bits get corrupted (probability p^3) or two of the three bits get corrupted (probability $3p^2$). Therefore probability of error = $p^3 + 3p^2$.

5

chapter

Transmission System Engineering

5.1 The output power after 10 km = −20 dBm and the power after 20 km = −23 dBm. This implies that the loss due to 10 km of fiber is 3 dB, or that the fiber loss = 0.3 dB/km. If the output power of the source is P_{in} dBm, we have

$$
\begin{aligned}
P_{in} &- 3 \text{ dB (source-fiber coupling loss)} \\
&- 3 \text{ dB (fiber loss)} \\
&- 1 \text{ dB (fiber detection coupling loss)} \\
&= -20 \text{ dBm}, \\
\text{or } P_{in} &= -13 \text{ dBm} = 50\ \mu\text{W}.
\end{aligned}
$$

5.2 (a)

$$\text{Loss limit} = \frac{\left(-3 \text{ dBm} + 30 \text{ dBm}\right)}{0.25 \text{ dB/km}} = 108 \text{ km}.$$

$$\text{Dispersion limit} = \frac{500 \text{ ps}}{17 \text{ ps/km-nm}} \frac{1}{1 \text{ nm}} = 29.4 \text{ km}.$$

Therefore, the longest link length is 29.4 km.

(b)

$$\text{Loss limit} = \frac{0 \text{ dBm} + 30 \text{ dBm}}{0.5 \text{ dB/km}} = 60 \text{ km}.$$

The dispersion limit is infinite. Therefore the longest link length is 60 km.

(c)

$$SNR = \frac{(G_m \mathcal{R} P_{in})^2}{2eG_m^2 F_A(G_m)\mathcal{R} P_{in} B_e + \frac{4k_B T}{R_L} F_n B_e}.$$

We have $G_m\mathcal{R} = 8$ A/W, $G_m = 10$ (hence $\mathcal{R} = 0.8$ A/W), $F_A = 3.16$ (5 dB), $F_n = 2$ (3 dB), $R_L = 50\ \Omega$, and $SNR = 1000$ (30 dB). Assume $T = 300$°K. Substituting these values and further assuming that $B_e = 500$ MHz, we get

$$1000 = \frac{64 P_{in}^2}{4.045 \times 10^{-8}\ P_{in} + 3.312 \times 10^{-13}}$$

or

$$64P_{in}^2 - 4.045 \times 10^{-5}\ P_{in} - 3.312 \times 10^{-10} = 0$$

or $P_{in} = 2.613\ \mu\text{W} = -25.8$ dBm. Thus, the longest link length $= 25.8/0.5 = 51.6$ km.

(d) Now we have

$$SNR = \frac{(\mathcal{R}P_{in})^2}{2e\mathcal{R}P_{in}B_e + \frac{4k_BT}{R_L}\ F_nB_e}.$$

Using $\mathcal{R} = 0.8$ A/W, $F_n = 3.16$ (5 dB), $R_L = 300\ \Omega$, $B_e = 500$ MHz, $T = 300°$K, and $SNR = 100$ (20 dB), we get

$$100 = \frac{0.64P_{in}^2}{1.28 \times 10^{-10}P_{in} + 8.72 \times 10^{-14}}$$

or

$$0.64P_{in}^2 - 1.28 \times 10^{-8}P_{in} - 8.72 \times 10^{-12} = 0$$

or $P_{in} = 3.7\ \mu\text{W} = -24.3$ dBm. Thus the longest link length $= 24.3/0.5 = 48.6$ km.

5.3 (a) We use $BL|D|\Delta\lambda < 0.491$ for a 2 dB penalty, since the source spectral width (10 nm) is large compared to the bit rate.

$$\begin{aligned} B &= 100 \text{ Mbps} &\Rightarrow L < 28.9 \text{ km}. \\ &= 1 \text{ Gbps} &\Rightarrow L < 2.89 \text{ km}. \\ &= 10 \text{ Gbps} &\Rightarrow L < 289 \text{ m}. \end{aligned}$$

(b) We use $BL|D|\Delta\lambda < 0.491$ for $B = 100$ Mbps and 1 Gbps since the spectral width 1 nm $\approx$ 120 GHz is large compared to the bit rate in these cases.

$$\begin{aligned} B &= 100 \text{ Mbps} &\Rightarrow L < 2890 \text{ km}. \\ &= 1 \text{ Gbps} &\Rightarrow L < 289 \text{ km}. \end{aligned}$$

The same formula for $B = 10$ Gbps yields, $L < 28.9$ km. If we use the small spectral width formula

$$B\lambda\sqrt{\frac{|D|L}{2\pi c}} < 0.491,$$

we get $L < 111$ km. The actual limit will be somewhere between the two.

(c) For $B = 100$ Mbps, the large spectral width formula applies and $L < 28,900$ km.

For $B = 1$ Gbps the spectral width is comparable to the modulation bandwidth. The large spectral width formula yields $L < 2890$ km, whereas the small spectral width formula yields $L < 11{,}100$ km. The actual limit will be between these two.

For $B = 10$ Gbps, the small spectral width formula applies and yields $L < 111$ km.

5.4 We use the same reasoning as in Problem 5.3.

(a)

B	$L <$
100 Mbps	98.3 km
1 Gbps	9.83 km
10 Gbps	983 m

(b)

B	$L <$	
100 Mbps	9830 km	
1 Gbps	983 km	
10 Gbps	98.3 km	(377 km using small spectral width formula)

(c)

B	L <	
100 Mbps	98,300 km	
1 Gbps	9830 km	(37,700 km using small spectral width formula)
10 Gbps	377 km	

5.5 (a) Left to the reader.

(b) Note that the effective index of the InGaAsP material used in the DFB laser is not specified. It is approximately 3.5. Using this value, the period of the grating is given by $\Lambda = \lambda_0/2n_{eff} = 1310/3.5 = 374.3$ nm.

(c) Note that NA is not defined in the book. Using (2.2), the NA is defined as

$$NA = \sin\theta_0^{\max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}.$$

The NA for this fiber is therefore 0.173, assuming $n_0 = 1$, which corresponds to a critical angle $\theta_0^{\max} = 10$ degrees.

(d) Using (2.3), the intermodal dispersion limited transmission length is given by

$$L = \frac{1.49 \times 3 \times 10^8}{2 \times 155.52 \times 10^6 \times 1.5^2 \times 0.01/1.5} = 96 \text{ m}.$$

(e) Using a loss of 0.4 dB/km yields a total link loss of 0.04 dB.

(f) The received power $P = -0.04$ dBm $= 0.9$ mW. From Section 3.6.1, the photocurrent, with wavelength expressed in microns, is given by

$$frac\lambda 1.24P \text{ A/W} = 1 \text{ mA}.$$

(g) For the fiber to be single-moded at 1310 nm, from (2.12), we need the fiber core radius

$$a < \frac{2.405\lambda}{2\pi\sqrt{n_1^2 - n_2^2}} = 2.9\ \mu\text{m}.$$

Therefore the core diameter needs to be smaller than 5.8 μm.

5.6 (a) Left to the reader.

(b) The wavelengths are 1550.918, 1551.721, 1552.524, 1553.329, and 1554.134 nm.

(c) The total launch power is 5 mW so the power per channel is 1 mW.

(d) From (5.15), the chromatic dispersion limit for 1 dB penalty is

$$L = 0.306/(2.5 \text{ Gb/s} \times 0.1 \text{ nm} \times 17 ps/nm - km) = 72 \text{ km}.$$

For PMD, using (5.23), we must have

$$L = \left(\frac{0.1 \times 400 \text{ ps}}{0.5 \text{ ps}/\sqrt{\text{km}}}\right)^2 = 6400 \text{ km}.$$

To compute the loss limit, we need to assume a particular receiver sensitivity and wavelength demultiplexer loss. Assuming a sensitivity of −30 dBm for the receiver (see Figure 4.9) and a loss of 5 dB for the demultiplexer, the allowable link budget, assuming no additional margins are required, is 32 dB, which translates into a link length of 128 km.

(e) The limiting factor is chromatic dispersion, and the allowed link length is 72 km.

5.7 (a) Left to the reader.

(b) Since the fiber has zero dispersion at 1310 nm, the link is loss limited, not chromatic dispersion limited.

(c) From Section 4.4.1, we have, for an ideal quantum limited receiver,

$$BER = 0.5e^{-M}$$

where M is the average number of photons received during a 1 bit. We need $M = 27$ for a BER of 10^{-12}. The corresponding average power, including 0 bits, is

$$\frac{1}{2}hfMB = 0.5*6.63\times10^{-34}\times\frac{3\times10^8}{1310\times10^{-9}\times27\times2.5\times10^9} = 5.12\times10^{-6}\text{ mW} = -53\text{ dBm}.$$

(d) The average photocurrent is given by

$$\frac{e}{hf}P = 5.4\times10^{-6}\text{ mA}.$$

(e) Since the link loss is 24 dB, we would need a launch power of $-53+24=-29$ dBm.

5.8 (a) Left to the reader.

(b) Since there are two additional 3-dB couplers in the path, the launch power needs to be increased to -23 dBm.

(c) At 2.5 Gb/s, the sensitivity at 1550 nm can be calculated as in Problem 5.7 to be 4.33×10^{-6} mW $=-53.6$ dBm. At 10 Gb/s, the sensitivity is 17.32×10^{-6} mW $=-47.6$ dBm.

(d) The PMD limit is independent of the wavelength. Using (5.23), the limiting bit rate for both systems is given by

$$B < \frac{0.1}{D_{PMD}\sqrt{L}} = \frac{0.1}{1\times10^{-12}\sqrt{60}} = 12.9\text{ Gb/s}.$$

The chromatic dispersion limited bit rate for the 1550 nm channel is given from (5.15) as

$$B < \frac{0.306}{DL\Delta\lambda} = \frac{0.306}{17\text{ ps/nm-km}\times60\text{ km}\times0.1\text{ nm}} = 3\text{ Gb/s}.$$

The loss limit depends on the launch power used.

(e) 10 Gb/s cannot be transported in the new system because of the chromatic dispersion limitation.

(f) At 2.5 Gb/s, for the 1550 nm channel, we need a minimum launch power of $-53.6+0.25\times60+6=-32.6$ dBm.

5.9

$$\text{PP} = -10\log\left(\frac{\frac{\mathcal{R}(P_1'-P_0')}{\sigma_1'+\sigma_0'}}{\frac{\mathcal{R}(P_1-P_0)}{\sigma_1+\sigma_0}}\right)$$

If we assume $P_1 \gg P_0,\ P_1' \gg P_0',\ \sigma_1' \gg \sigma_0', \sigma_1 \gg \sigma_0$ and $\sigma_1\alpha\sqrt{P_1},\ \sigma_1'\alpha\sqrt{P_1'}$, we get

$$\text{PP}_{\text{sig-indep}} = -10\log\left(\frac{\sqrt{P_1'}}{\sqrt{P_1}}\right) = -5\log\frac{P_1'}{P_1}.$$

5.10

$$\text{PP} = -10\log\left(\frac{\frac{\mathcal{R}(P_1'-P_0')}{\sigma_1'+\sigma_0'}}{\frac{\mathcal{R}(P_1-P_0)}{\sigma_1+\sigma_0}}\right).$$

With an ideal extinction ratio, we have $P_1 = 2P$, $P_0 = 0$, $\sigma_1^2 = 2xP + y$, and $\sigma_0^2 = y$, where $x = 4\mathcal{R}^2 G P_n(G-1)B_e$ and $y = 2\mathcal{R}^2[P_n(G-1)]^2(2B_o - B_e)B_e$. Here we have considered only signal-spontaneous and spontaneous-spontaneous beat noise.

With an extinction ratio of r, we have (see Section 5.3, p. 207), $P_1' = 2rP/(r+1)$, $P_0' = 2P/(r+1)$, $\sigma_1'^2 = 2xrP/(r+1) + y$, and $\sigma_0'^2 = 2xP/(r+1) + y$.

Therefore,

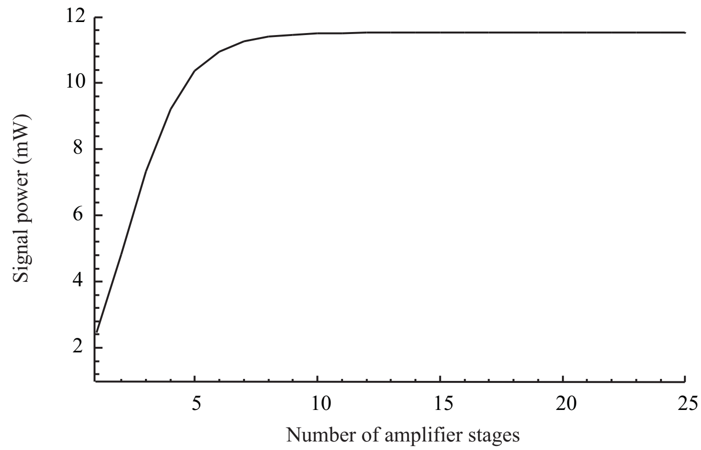
$$\begin{aligned}
\text{PP} &= -10\log\left(\frac{\frac{2P\mathcal{R}(r-1)/(r+1)}{\sqrt{2xrP/(r+1)+y}+\sqrt{2xP/(r+1)+y}}}{\frac{2P\mathcal{R}}{\sqrt{2xP+y}+\sqrt{y}}}\right) \\
&= -10\log\left(\frac{r-1}{r+1}\frac{\sqrt{2xP+y}+\sqrt{y}}{\sqrt{2xrP/(r+1)+y}+\sqrt{2xP/(r+1)+y}}\right).
\end{aligned}$$

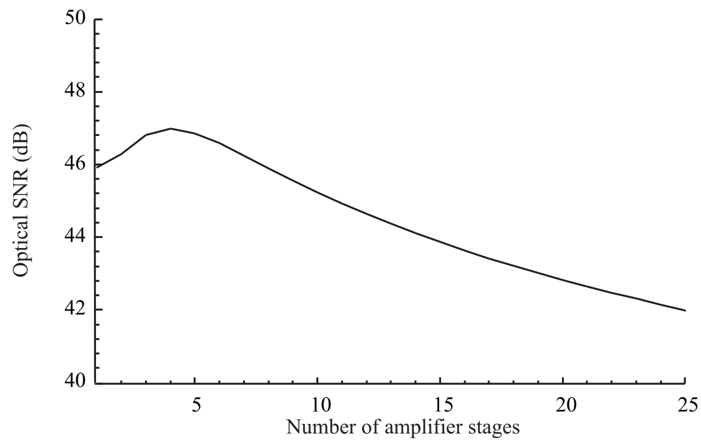
If $y \ll 2xP/(r+1)$, that is, the spontaneous-spontaneous noise term can be neglected in comparison with the signal-spontaneous term, even for a 0 bit (in the nonideal extinction ratio case), this expression simplifies to

$$\text{PP} = -10\log\left(\frac{r-1}{r+1}\frac{\sqrt{r+1}}{\sqrt{r}+1}\right).$$

5.11 Solving equations (5.6) and (5.7) with the given values of the other parameters ($G_{\max} = 35$ dB, $l = 120$ km, $\alpha = 0.25$ dB/km, $n_{sp} = 2$, $P^{sat} = 10$ mW, and $B_o = 50$ GHz), we get, $\overline{P}_{out} = 11.524$ mW and $\overline{G} = 999.998$. Since the loss between stages is $0.25 \times 120 = 30$ dB, or 1000, the steady-state gain is slightly smaller, as expected. The steady-state amplifier output power (11.5 mW) is somewhat larger than its internal saturation power (10 mW).

We assume that a signal with an input power of 1 mW is transmitted. The evolution of the signal power and optical SNR, at the output of each amplifier, are plotted below.





Note that the signal power reaches its steady state value of 11.5 mW calculated above, after a few stages. The optical SNR increases for the first few stages but later decreases with increasing number of stages, due to accumulation of noise at each stage.

5.12 Using $P_0' = \epsilon P$, $\sigma_0' \propto \sqrt{\epsilon P}$, $P_1' = P(1 - 2\sqrt{\epsilon})$, and $\sigma_1' \propto \sqrt{P_1'}$, we get,

$$\frac{P_1' - P_0'}{\sigma_1' + \sigma_0'} = \frac{1 - 2\sqrt{\epsilon} - \epsilon}{\sqrt{\epsilon} + \sqrt{1 - 2\sqrt{\epsilon}}}\sqrt{P}.$$

Using $\sqrt{1-2\sqrt{\epsilon}} = 1 - \sqrt{\epsilon} + O(\epsilon)$, the $\sqrt{\epsilon}$ terms in the denominator cancel and we get the denominator is $1 + O(\epsilon)$. Neglecting the $O(\epsilon)$ terms, and using this along with $\sigma_1 \propto \sqrt{P}$ in (5.2), we get (5.12).

5.13

$$E(t) = \sqrt{2} P d_s(t) \cos[\omega_c t + \phi_s(t)] + \sum_{i=1}^{N} \sqrt{2\epsilon_i}\, d_{xi}(t) \cos\,[\omega_c t + \phi_{xi}(t)]\,.$$

The received power is proportional to the square of the electric field and is thus given by

$$P_r = P d_s(t) \quad + \quad \sum_{i+1}^{N} \epsilon_i d x_i(t) + \sum_{i=1}^{N} 2\sqrt{\epsilon_i}\, P d_{xi}(t) \cos[\phi_s(t) - \phi_{xi}(t)]$$

$$+ \quad \sum_{i=1}^{N} \sum_{j=1}^{N} 2\sqrt{\epsilon_i \epsilon_j} P d_{xi}(t) d_{xj}(t) \cos[\phi_{xi}(t) - \phi_{xj}(t)].$$

Neglecting the $\sqrt{\epsilon_i \epsilon_j}$ term, we get (5.9)–(5.12) with $\sqrt{\epsilon} = \sum_{i=1}^{N} \sqrt{\epsilon_i}$ in (5.9) and (5.10) and $\epsilon = \sum_{i=1}^{N} \epsilon_i$ in (5.11) and (5.12).

5.14 Equation (5.16) is an approximation because $\frac{L}{l}$ may not be an integer.

A precise form of this equation is

$$L_e = \frac{1 - e^{\alpha l}}{\alpha} \left\lfloor \frac{L}{l} \right\rfloor + \frac{1 - e^{-\alpha(L - \lfloor \frac{L}{l} \rfloor l)}}{\alpha}.$$

This equation is derived by observing that when amplifiers are placed l km apart, there are $\lfloor \frac{L}{l} \rfloor$ amplifiers in a link of length L. Adding the contributions from these $\lfloor \frac{L}{l} \rfloor$ spans gives the first term. The second term is the effective length of the remaining link length, namely, $L - \lfloor \frac{L}{l} \rfloor l$.

5.15 Let $\delta = 10^{-C/10}$. Since the crosstalk adds coherently, (5.9) applies if we assume detection limited by thermal noise.

(a) For coherent addition of crosstalk in N stages, the crosstalk level after N nodes is $(N\sqrt{\delta})^2$.

(b) After 5 nodes, the crosstalk level is $(5\sqrt{\delta})^2 = 25\delta$. The crosstalk penalty is is given by

$\text{PP} = -10 \log (1 - 2\sqrt{25\delta})$.

For a 1 dB penalty, $C = 33.7$ dB.

5.16 Assume the crosstalk power from each adjacent channel is $\frac{\epsilon}{2}P$ and the crosstalk power from non-adjacent channels is negligible. Then

$$P_0 = \frac{2P}{r+1} \text{ and } P_1 = \frac{2rP}{r+1} \text{ where } P = \frac{P_0 + P_1}{2} \text{ where} \frac{P_1}{P_0} = r.$$

In the worst case,

$$P'(1) = \frac{2rP}{r+1} + \epsilon \frac{2P}{r+1} = \frac{2P}{r+1}(r + \epsilon) \text{ (adjacent channels send a 0 bit)},$$

and

$$P'(0) = \frac{2P}{r+1} + \epsilon \frac{2rP}{r+1} = \frac{2P}{r+1}(1 + \epsilon r) \text{ (adjacent channels send a 1 bit)}.$$

So we get

$$\text{PP}_{\text{sig-indep}} = -10 \log \left(\frac{r + \epsilon - 1 - \epsilon r}{r+1} \right) = -10 \log \left[\frac{(r-1)(1-\epsilon)}{r+1} \right].$$

For $r = 10$ (10 dB extinction ratio), $\epsilon = -15.35$ dB. Therefore, for each adjacent channel, the crosstalk supression should be -18.35 dB.

5.17 (a) Let C dB correspond to a fraction δ, that is, $\delta = 10^{-C/10}$. After demultiplexing, a fraction δ of the power from say, wavelength i, is present in the adjacent channels $i+1$ and $i-1$.

After multiplexing, at the wavelength i, we get two crosstalk signals with powers δ^2 each added coherently for a total power of $4\delta^2$. We assume that the detection is limited by thermal noise, so that (5.9) applies. In this case we have $\epsilon = 4\delta^2$ for each stage and after N stages,

$$\sqrt{\epsilon} = N\sqrt{4\delta^2}.$$

Thus

$$\begin{aligned} \text{PP} &= -10\log(1-2\sqrt{\epsilon}) \\ &= -10\log(1-20\delta) \text{ for } N=5 \\ &= -10\log(1-20\times 10^{-C/10}). \end{aligned}$$

(b)

$$\begin{aligned} \text{PP} &< 1 \text{ dB} \Rightarrow 1-20\times 10^{-C/10} < 10^{-0.1}, \\ &\text{or} \quad 10^{-C/10} > 0.0103, \\ &\text{or} \quad C > 19.9 \text{ dB}. \end{aligned}$$

5.18 The crosstalk from the mux/demuxes due to each adjacent channel is $2\times(-25) = -50$ dB below the desired signal. However, this is intrachannel crosstalk as is the crosstalk of -40 dB from the switch. Thus there are three crosstalk signals with $\epsilon_1 = \epsilon_2 = 10^{-50/10}$, and $\epsilon_3 = 10^{-40/10}$ at each stage.

After N stages, there are $3N$ crosstalk signals with $2N$ of them 50 dB below and N of them 40 dB below the signal. Therefore,

$$\begin{aligned} \epsilon &= \left(N\sum_{i=1}^{3}\sqrt{\epsilon_i}\right)^2 \\ &= 2.665\times 10^{-4}N^2. \end{aligned}$$

For a 1-dB penalty,

$$\begin{aligned} -10\log(1-2\sqrt{\epsilon}) &< 1 \\ \text{or} \quad \sqrt{\epsilon} &< 0.103 \\ \text{or} \quad 0.0163N &< 0.103 \\ \text{or} \quad N &< 6.3. \end{aligned}$$

Therefore, six nodes can be cascaded in a network with a penalty < 1 dB for detection limited by thermal noise.

5.19 (a) The best case transmittance is when all the muxes and demuxes have their centre wavelength $\lambda'_c = \lambda_c$. The transmittance in this case is 1, or equivalently, 0 dB. The worst case transmittance occurs when all the muxes and demuxes have their center wavelengths at $\lambda_c + \Delta\lambda$ or $\lambda_c - \Delta\lambda$. This worst case transmittance is given by

$$\left[e^{-(\Delta\lambda)^2/2\sigma^2}\right]^N = (0.9692)^N = 0.1357N \text{ dB}.$$

(b) We need

$$\left[e^{-(\Delta\lambda)^2/2(0.2)^2}\right]^{10} = \frac{1}{2},$$

which yields $\Delta\lambda = 0.0745$ nm.

5.20 From each adjacent channel, the crosstalk power in one stage for 0.8 nm separation is given by

$$e^{-(0.8)^2/2(0.2)^2} = 3.3546 \times 10^{-4} = -34.74 \text{ dB}.$$

Thus after N stages the crosstalk power $= 31.73N$ dB. When the adjacent channels are at the worst-case positions, the crosstalk power from both adjacent channels is given by

$$2 \times e^{-(0.75)^2/2(0.2)^2} = -27.53 \text{ dB}.$$

After N stages, the crosstalk power $= 27.53N$ dB.

5.21 The added wavelength undergoes a loss of
1 dB (new circulator)
+ 20 dB (grating transmission of 1% corresponding to reflectivity of 99%)
+ 1 dB ("drop" circulator)
= 22 dB.

So the crosstalk power from leakage of the added wavelength into the dropped wavelength = 0 dBm − 22 dB = −22 dBm.

The loss undergone by the dropped wavelength is
1 dB ("drop" circulator)
+ 20 dB (grating)
+ 1 dB ("add" circulator)
= 22 dB.

Thus the crosstalk power from the dropped wavelength into the added wavelength = −30 dBm − 22 dB = −52 dBm, which is small compared to the power of the added signal.

However, the crosstalk power from the added signal (−22 dBm) into the dropped signal is much larger than the power of the dropped signal itself, which is
−30 dBm
− 1 dB (first circulator pass)
− 0.04 dB (grating)
− 1 dB (second circulator pass)
= −32 dBm.
Therefore the element will not work.

5.22 From the solution of Problem 5.30,

$$T_{0,\text{opt}} = (1+\kappa^2)^{1/4}\sqrt{|\beta_2|L}.$$

Setting $\kappa = 0$ for an unchirped pulse, we get

$$T_{0,\text{opt}} = \sqrt{|\beta_2|L}.$$

5.23

$$\mathrm{PP}(dB) = \alpha \frac{\Delta\tau^2}{T^2}\epsilon(1-\epsilon).$$

Denote the probability density function of $\Delta\tau$ by $f_{\Delta\tau}(.)$ and its (cumulative) distribution function by $F_{\Delta\tau}(.)$.

$$\begin{aligned}
\Pr(\mathrm{PP} < p) &= \Pr\left(\Delta\tau^2\epsilon(1-\epsilon) \le x = T^2 p/\alpha\right) \\
&= \int_{t=0}^{\infty} \Pr\left(\epsilon(1-\epsilon) \le x/\Delta\tau^2 | \Delta\tau = t\right) f_{\Delta\tau}(t)\, dt \\
&= \int_{t=0}^{\infty} \Pr\left(\left(\epsilon - 0.5 + \sqrt{0.25 - x/t^2}\right)\left(\epsilon - 0.5 - \sqrt{0.25 - x/t^2}\right) > 0\right) f_{\Delta\tau}(t)\, dt
\end{aligned}$$

$$\begin{aligned}
\Pr\left(\left(\epsilon - 0.5 + \sqrt{0.25 - x/t^2}\right)\left(\epsilon - 0.5 - \sqrt{0.25 - x/t^2}\right) > 0\right) &= 1, \text{ for } t^2 < 4x, \\
&= 1 - \sqrt{1 - 4x/t^2}, \text{ for } t^2 \ge 4x.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Pr\left(\mathrm{PP} < p = x\alpha/T^2\right) &= F_{\Delta\tau}\left(2\sqrt{x}\right) + \int_{t=2\sqrt{x}}^{\infty}\left(1 - \sqrt{1 - 4x/t^2}\right) f_{\Delta\tau}(t)\, dt \\
&= 1 - \int_{t=2\sqrt{x}}^{\infty} \sqrt{1 - 4x/t^2}\, f_{\Delta\tau}(t)\, dt.
\end{aligned}$$

Using (see Appendix H.1.2),

$$f_{\Delta\tau}(x) = \frac{\sqrt{2}}{a^3\sqrt{\pi}} x^2 e^{-x^2/2a^2}, \qquad x \ge 0,$$

and the relation (use a symbolic integration package such as Mathematica™ or see a table of integrals),

$$\int_{t=y}^{\infty} \frac{\sqrt{t^2 - y^2}}{t} f_{\Delta\tau}(t)\, dt = e^{-y^2/2a^2},$$

we get,

$$\Pr\left(\mathrm{PP} < p = x\alpha/T^2\right) = 1 - e^{-4x/2a^2} = 1 - e^{-4pT^2/2\alpha a^2}.$$

Therefore, PP is exponentially distributed with mean $\alpha a^2/2T^2$. Using $\langle\Delta\tau\rangle = 2a\sqrt{2/\pi}$ (Appendix H.1.2) or $a = \langle\Delta\tau\rangle\sqrt{\pi/8}$, the mean of PP is $\pi\alpha\langle\Delta\tau\rangle^2/16T^2$.

$$\Pr(\mathrm{PP} \ge 1) = e^{-\frac{16T^2}{\pi\alpha\langle\Delta\tau\rangle^2}}.$$

Assuming $\alpha = 16$ and $\langle \Delta\tau \rangle = 0.3T$, $\Pr(\text{PP} \geq 1) \approx 0.03$. Thus, if the average DGD is less than $0.3T$, the power penalty due to PMD is unlikely to exceed 1 dB.

5.24

$$\begin{aligned} \frac{dI_s}{dz} &= -g_B I_p I_s + \alpha I_s. && (5.17) \\ \frac{dI_p}{dz} &= -g_B I_p I_s - \alpha I_p. && (5.18) \end{aligned}$$

Neglecting the depletion of the pump wave, (5.18) becomes

$$\frac{dI_p}{dz} = -\alpha I_p.$$

Solving this equation, we get

$$I_p(z) = I_p(0)e^{-\alpha z}.$$

Substituting this in (5.17) yields

$$\begin{aligned} \frac{dI_s}{dz} &= -g_B I_p(0)e^{-\alpha z} I_s + \alpha I_s \\ &= (\alpha - g_B I_p(0)e^{-\alpha z}) I_s, \end{aligned}$$

or,

$$\frac{dI_s}{dz} + (g_B I_p(0)e^{-\alpha z} - \alpha) I_s = 0.$$

Solving, we get

$$e^{-\alpha z} e^{g_B I_p(0)\frac{e^{-\alpha z}}{-\alpha}} \; I_s(z) = \text{constant}, \; c.$$

Setting $z = 0$, we obtain

$$c = e^{-g_B I_p(0)/\alpha} \; I_s(0).$$

Therefore,

$$\begin{aligned} I_s(z) &= e^{\alpha z} \, e^{\frac{-g_B I_p(0)}{\alpha}[1 - e^{-\alpha z}]} I_s(0) \\ \text{or } I_s(L) &= e^{\alpha L} \, e^{\frac{-g_B I_p(0)}{\alpha}[1 - e^{-\alpha L}]} I_s(0). \end{aligned}$$

Recognizing that $\frac{1 - e^{-\alpha L}}{\alpha} = L_e$ and using $P_p = A_e I_p$ and $P_s = A_e I_s$ we get

$$P_p(L) = P_p(0)e^{-\alpha L}$$

and

$$P_s(0) = P_s(L)e^{-\alpha L} \; e^{\frac{g_B P_p(0) L_e}{A_e}}.$$

5.25 As the SBS interaction occurs within a single wavelength, it does not matter whether the system has one or many channels.

(a) Only the line width of a single line matters since the line separation is much greater than the SBS gain bandwidth Δf_B of 20 MHz. The SBS threshold power

$$P_{\text{th}} \approx \frac{21bA_e}{g_B L_e}\left(1+\frac{\Delta f_{\text{source}}}{\Delta f_B}\right).$$

Assuming $b = 1$,

$$P_{\text{th}} \approx 1.3\left(1+\frac{\Delta f_{\text{source}}}{\Delta f_B}\right) \text{ mW}.$$

For $\Delta f_{\text{source}} = 1$ GHz and $\Delta f_B = 20$ MHz, $P_{\text{th}} = 66$ mW.

(b) Again $P_{\text{th}} \approx 1.3\left(1+\frac{1000}{20}\right) \approx 66$ mW.

(c) $P_{\text{th}} \approx 1.3\left(1+\frac{10000}{20}\right) \approx 650$ mW.

5.26 (a) From (5.27),

$$\frac{T_L}{T_0} = \sqrt{1+\sqrt{2}\frac{L_e}{L_{NL}}\frac{L}{L_D}+\left(1+\frac{4}{3\sqrt{3}}\frac{L_e^2}{{L_{NL}}^2}\right)\frac{L^2}{L_D^2}}.$$

Therefore,

$$T_L^2 = T_0^2 + \sqrt{2}\frac{L_e}{L_{NL}}L\beta_2 + \left(1+\frac{4}{3\sqrt{3}}\frac{L_e^2}{{L_{NL}}^2}\right)\frac{\beta_2^2 L^2}{T_0^2}.$$

Denoting, $\kappa_{NL}^2 = \frac{4}{3\sqrt{3}}\frac{L_e^2}{{L_{NL}}^2}$, the optimum T_0 satisfies (see solution to problem 5.30),

$$\frac{\partial T_L^2}{\partial T_0} = 0$$

or $T_{0,\text{opt}}(L) = \sqrt{|\beta_2|L}(1+\kappa_{NL}^2)^{1/4}$.

(b) Denoting $\alpha = \sqrt{2}L_e/L_{NL}$ (which is also proportional to κ_{NL}), the optimum final pulse width $T_{L,\text{opt}}$ is obtained by solving

$$T_{L,\text{opt}}^2 = T_{0,\text{opt}}^2 + \alpha L\beta_2 + \left(1+\kappa_{NL}^2\right)\frac{\beta_2^2 L^2}{T_{0,\text{opt}}^2},$$

as

$$T_{L,\text{opt}}(L) = \sqrt{|\beta_2|L}\sqrt{2\sqrt{1+\kappa_{NL}^2}+\text{sgn}(\beta_2)\alpha}.$$

Note the similarity to $T_{L,\text{opt}}$ in the solution of Problem 5.30. In fact, by setting $\kappa_{NL} = \kappa$ and $\alpha = 2\kappa$ in the above expression, we get the expression for $T_{L,\text{opt}}$ in Problem 5.30, as we expect.

We assume that satisfactory communication is possible with a power penalty PP(ϵ) if the width of the pulse as measured by its rms width T^{rms} is less than $1+\epsilon$ times the bit period. Therefore, we must have, $T_L < \sqrt{2}(1+\epsilon)/B$. The maximum link length for which the output pulse has an rms width less than $(1+\epsilon)$ times the bit period is given by the solution of

$$T_{L,\text{opt}}^2(L) = 2(1+\epsilon)^2/B^2$$

which is

$$L_{\max} = \frac{2(1+\epsilon)^2}{B^2|\beta_2|\sqrt{2\sqrt{1+\kappa_{NL}^2}+\text{sgn}(\beta_2)\alpha}}.$$

We further assume, somewhat arbitrarily but in analogy with the NRZ pulse case, that $\epsilon = 0.306$ for a power penalty of 1 dB, that is, PP(0.306) = 1 dB. We can now calculate

$L_{\max}$ based on the other system parameters. E.g., for $B = 10$ Gb/s, $D = 17$ ps/km-nm, $\lambda = 1.55\ \mu$m, $L_e = 20$ km and $L_{NL} = 38.4$ km (which corresponds to a transmit power of 10 mW; see page 89), we get, $L_{\max} = 1077$ km , $T_{0,\mathrm{opt}}(L_{\max}) = 160$ ps, and $T_{L,\mathrm{opt}}(L_{\max}) = 185$ ps.

(c) If only chromatic dispersion were present, using (2.13), the output pulse width at the end of a link of length $L_{\max}$ when the input pulse width is $T_{0,\mathrm{opt}}$ is given by

$$\begin{aligned} T_{\mathrm{disp}}(L_{\max})^2 &= T_{0,\mathrm{opt}}(L_{\max})^2 + \beta_2^2 L_{\max}^2 / T_{0,\mathrm{opt}}(L_{\max})^2 \\ &= |\beta_2| L_{\max} \left(\sqrt{1+\kappa_{NL}^2} + \frac{\mathrm{sgn}(\beta_2)}{\sqrt{1+\kappa_{NL}^2}} \right). \end{aligned}$$

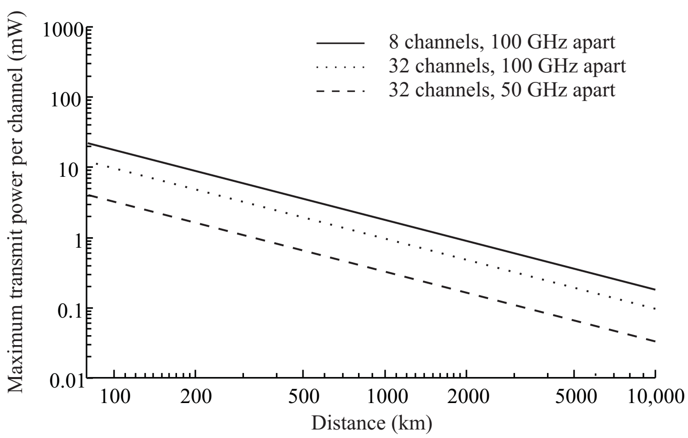
We can calculate the pulse broadening factor ϵ_{disp} due to dispersion using $T_{\mathrm{disp}}^2 = 2(1+\epsilon_{\mathrm{disp}})^2/B^2$ and estimate the power penalty due to dispersion alone by interpolation, using the values PP(0) = 0 dB, PP(0.306) = 1 dB and PP(0.491) = 2 dB. For the same values as in (b), $T_{\mathrm{disp}}(L_{\max}) = 216$ ps and $\epsilon_{\mathrm{disp}} = 0.53$ and we estimate that the power penalty due to dispersion alone to be 2.2 dB. Thus, in this example, the SPM penalty is -1.2 dB.

In general, the SPM penalty calculated in this way is negative if $D > 0$, that is, $\beta_2 < 0$ and positive otherwise.

Interestingly, we observe from the expressions for $T_{L,\mathrm{opt}}$ and T_{disp} that their ratio, and hence the SPM penalty calculated as above, depends only on L_e/L_{NL} (through κ_{NL} and α) and the sign of D (or β_2), and is independent of the actual value of D.

5.27 Using a computer program, a set of wavelengths with this property is 193.1, 193.3, 193.6 and 194.0 THz.

5.28



5.29 Second order nonlinearities typically have no effect on a lightwave system since the resulting frequencies $(f_1 + f_2)$ and $(f_1 - f_2)$ are out of band as long as the set of frequencies $f_1, f_2, \ldots, f_N$ all lie within a single octave, which is usually the case. In any event, the second order susceptibilities in silica are negligible.

5.30 The frequency spectrum of the source is given by

$$F(\omega) = B_0 \omega_0 e^{-(\omega-\omega_0)^2/2\omega_0^2},$$

where we have assumed a Gaussian profile. The *rms* spectral width is

$$\omega^{rms} = \frac{\omega_0}{\sqrt{2}}$$

(see solution to (2.10)). The 20-dB spectral width is given by $2(\omega_{20} - \omega_0)$ where ω_{20} solves

$$e^{-(\omega_{20}-\omega_0)^2/\omega_0^2} = 0.01.$$

We have used ω_0^2 instead of $2\omega_0^2$ in the exponent since the pulse power is proportional to the square of its amplitude. Solving this equation yields

$$2(\omega_{20} - \omega_0) = \sqrt{-4\ln 0.01}\omega_0.$$

Therefore

$$\frac{\text{20-dB spectral width}}{6.07} = \frac{\sqrt{-4\ln 0.01}}{6.07}\,\omega_0 = 0.707\omega_0 = rms \text{ spectral width.}$$

5.31 From (2.25),

$$T_L = \sqrt{\left(T_0 + \frac{\kappa\beta_2 L}{T_0}\right)^2 + \left(\frac{\beta_2 L}{T_0}\right)^2}.$$

The optimum T_0 satisfies

$$\begin{aligned}
&\tfrac{\partial T_L^2}{\partial T_0} = 0\\
&\Rightarrow 2\left(T_0 + \tfrac{\kappa\beta_2 L}{T_0}\right)\left(1 - \tfrac{\kappa\beta_2 L}{T_0^2}\right) + \tfrac{2\beta_2 L}{T_0}\left(\tfrac{-\beta_2 L}{T_0^2}\right) = 0\\
&\Rightarrow (1+\kappa x)(1-\kappa x) = x^2 \text{ where } x = \tfrac{\beta_2 L}{T_0^2}\\
&\Rightarrow x^2 = \tfrac{1}{1+\kappa^2}\\
&\text{or } T_{0,\text{opt}} = \sqrt{|\beta_2|L}(1+\kappa^2)^{1/4}.
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
T_{L,\text{opt}} &= T_{0,\text{opt}}\sqrt{\left(1 + \frac{\text{sgn}(\beta_2)\kappa}{\sqrt{1+\kappa^2}}\right)^2 + \frac{1}{1+\kappa^2}}\\
&= \frac{T_{0,\text{opt}}}{\sqrt{1+\kappa^2}}\sqrt{\left(\sqrt{1+\kappa^2} + \text{sgn}(\beta_2)\kappa\right)^2 + 1}\\
&= \sqrt{|2\beta_2|L}\sqrt{\sqrt{1+\kappa^2} + \text{sgn}(\beta_2)\kappa}.
\end{aligned}$$

In the 1.55 μm band, $\beta_2 < 0$. Thus for $\kappa = -6$,

$$T_{L,\text{opt}} = \sqrt{24.166|\beta_2|L}.$$

The condition $BT_L^{rms} < \epsilon$ translates to

$$BT_{L,\text{opt}} < \epsilon\sqrt{2}$$

$$\text{or } B\sqrt{24.166|\beta_2|L} \quad < \quad \epsilon\sqrt{2}$$

$$\text{or } B\lambda\sqrt{\frac{12.08|D|L}{2\pi c}} \quad < \quad \epsilon.$$

For $D = 17$ ps/km-nm, $\lambda = 1.55$ μm, and $\epsilon = 0.491$ (2 dB penalty), we get $B^2L < 921.05$ (Gb/s)^2-km. For $B = 1$ Gb/s, $L < 921$ km.

For the same values of D, λ,and ϵ, we have from Figure 2.10 (and the accompanying explanation) that $B^2L < 11126$ (Gb/s)^2-km. Thus for $B = 1$ Gb/s, $L < 11{,}126$ km which is much higher.

6 chapter

First-Generation Optical Networks

6.1 (a) Optical channel layer and/or SONET path layer.

(b) This would be handled by the SONET line layer, not any of the optical layers.

(c) Again this would be done by the SONET section layer. However, we may have OEOs within the optical layer itself to regenerate the signal on a wavelength-by-wavelength basis if we have exhausted the optical system link budget. In this case, the OEOs may monitor the error rate as well, and this function would be part of the optical channel layer.

6.2

System	Loss	Range at 1550 nm	Range at 1310 nm
SR	0–7 dB	0–28 km	0–14 km
IR	0–12 dB	0–48 km	0–24 km
LR	10–24 dB	40–96 km	20–48 km

6.3 The link from the S-16.1 transmitter to the I-16 receiver, we have:
Tx power: 0 to -5 dBm,
loss: 0 to 7 dB, and
Rx power: -3 to -18 dBm.
The maximum power received is 0 dBm and happens when the transmit power is 0 dBm and the loss is 0 dB. This is 3 dB larger than the receive overload value and hence in this case, a VOA with a range of 3 dB is needed.

For the link from the I-16 transmitter to the S-16-1 receiver, we have:
Tx power: -3 to -10 dBm,
loss: 0 to 7 dB, and
Rx power: 0 to -18 dBm.
The maximum power received is −3 dBm and happens when the transmit power is −3 dBm and the loss is 0 dB. This is less than the receive overload value and hence in this case, no VOA is needed.

7 chapter

WDM Network Elements

7.1 (a) Let L denote loss between the two nodes in dB. Then power received on λ_1 at node B's input is $0-L$ dBm. Minimum power on desired wavelength is -30 dBm. To get a crosstalk of 15 dB, assuming a suppression of S dB, we must have

$$-L - S \leq -45 \text{ dB}.$$

In a worst-case scenario $L = 0$ dB, in which case, we need $S = 45$ dB.

(b) Assume that 0 dBm is input to node A at the dropped wavelength. If T is the intrachannel crosstalk suppression, then the crosstalk power exiting node A is $-T$ dBm. With the signal at 0 dBm, we must have

$$0 - T \leq -30 \text{ dB},$$

or $T \geq 30$ dB, independent of the link loss.

7.2 A simple wavelength assignment for the lightpaths is AB, BC, and CD at λ_1; AC at λ_2; and BD at λ_3. Then node A drops/adds λ_1, λ_2; node B drops/adds λ_1, λ_3; node C drops/adds λ_1, λ_2; and node D drops/adds λ_1, λ_3.

For the new lightpaths, one possible wavelength assignment is AB, BC, and CD at λ_1; AD at λ_2; and BC at λ_3. Then node A drops/adds λ_1, λ_2; node B drops/adds λ_1, λ_3; node C drops/adds λ_1, λ_3; and node D drops/adds λ_1, λ_2.

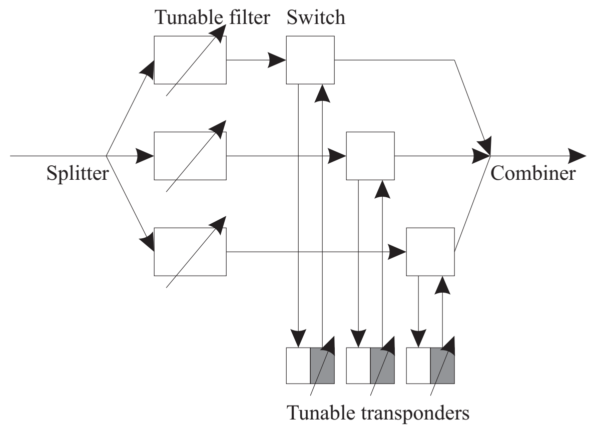
Note that nodes C and D have changed from before.

7.3 (a) Left to the reader.

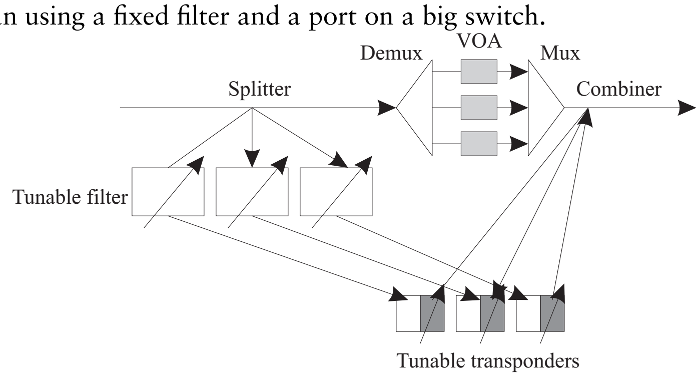
(b) With N intermediate OADMs, the total loss along the path is $2N + 2 + L$ where L denotes the total link loss. Therefore $2N + 2 + L \leq 30$.

(c) Left to the reader.

7.4 Consider the following OADM architecture.



The main difference between this architecture and that of Figure 7.7(d) lies in the use of tunable filters and small switches instead of a mux/demux and a big switch. Since splitters and combiners are used, there is a minimum passthrough loss of $20 \log W$, where W is the number of channels. So a 32-channel OADM will have a minimum passthrough loss of 32 dB, which is quite high. Also now a tunable filter is required for each wavelength, which may or may not be more expensive than using a fixed filter and a port on a big switch.



Another plausible OADM architecture is shown above. Here a wavelength blocker device (a demux/mux combination with a per-channel variable optical attentuator) is used to either block the add/drop channels from passing through as well as equalize power levels for the passthrough channels. The loss in the passthrough path is low, but the loss in the add/drop path is high due to the splitters and combiners. However, tunable filters need be provided only for drop channels and not for all channels.

7.5 Each remote node drops and adds 2 wavelengths and 8 wavelengths are needed in total. Hub drops and adds all wavelengths.

System 1: Remote node needs 1 OADM and 2 regenerators for a cost of \$40,000. Hub node requires 2 OADMs for a cost of \$40,000, so total network cost is \$200,000.

System 2: Remote node needs 2 OADMs for a cost of \$20,000. Hub node needs 8 OADMs at \$80,000. Total network cost including amplifiers is \$220,000.

7.6 (a) For each WDM system, we require 24 line ports on the OXC and 16 trib ports, or 40 ports. Therefore a 256-port OXC can support 6 WDM systems.

(b) Out of the 24 lightpaths passing through, 6 of them need to be converted, taking up a total of 12 additional OXC ports. Now we need 52 ports per WDM system. Therefore a 256-port OXC can support 4 WDM systems.

(c) The 24 lightpaths passing through take up only 6 OXC ports, and the 16 drop/adds take up 4 more ports for a total of 20 ports. Therefore a 256-port OXC can support 12 WDM systems.

7.7 (a) The figure is essentially Figure 7.15. However, since the tuning range is limited, the add/drop switch can be partitioned into a number of smaller switches, each switch being connected only to a subset of the passthrough wavelength plane switches. In this case, we will use 5 smaller switches, the first one connected to wavelength plane switches for λ_1 through λ_8, the second connected to the switchesfor λ_9 through $\lambda_1$7 etc.

The 4 add/drop channels may all be within a single band. So we need to pre-equip the node with 4 tunable lasers for each band, or a total of 20 tunable lasers.

(b) Since each laser now tunes over 2 bands, we can reduce the number of pre-equipped transponders but will need larger add/drop switches. Each add/drop switch needs to be connected to the wavelength plane switches for 2 bands. Say we decide to allocate a pool of lasers for bands 1 and 2, another pool for bands 3 and 4, and a third pool for band 5. Now we'll need to pre-equip 4 transponders for each pool, or a total of 12 transponders.

8 chapter

Control and Management

8.1 (a) Setting up and taking down lightpaths in the network: OCh
(b) Monitoring and changing the digital wrapper overhead in a lightpath: OCh
(c) Rerouting all wavelengths (except the optical supervisory channel) from a failed fiber link onto another fiber link: OMS
(d) Detecting a fiber cable cut in a WDM line system: OTS
(e) Detecting failure of an individual lightpath: OCh
(f) Detecting bit errors in a lightpath: OCh

8.2 Number the nodes from left to right. Node 2 is the amplifier for example. Assume that the regenerator is part of the SONET layer and that the connection is processed by each network element shown in the figure. The story would be different if the signal were for example bypassed through Node 3 optically without going to the SONET ADM. In this case, Node 3 would terminate layers up to the OMS only for this connection.

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Path					Path
Line		Line		Line	Line
Section		Section	Section	Section	Section
OC		OC	OC		
OMS		OMS	OMS		
OAS	OAS	OAS	OAS		

8.3

- Assume the fiber cut occurs at time 0, and at the middle of the link between A and B. Also assume time is an integer in millseconds. The following events would occur. 0: Fiber cut.
  1: Loss of light at node B.
  3=1+2: Node B detects loss of light.
  8=3+5: Node B transmits OMS-FDI to node E, and OTS-BDI to node A.
  11=8+3: OTS-BDI received by node A, if the fiber from B to A is not cut.
  17=8+3+3+3: OMS-FDI from B received by E.
  22=17+5: OCh-FDI transmitted by E for all lightpaths passing through it, for example,

those destined for G. 25=22+3: OCh-FDI received from E for lightpaths affected by the cut.
2003=3+2000: Alarm raised by node B.

- If FDI signals are sent immediately, the following would be the timeline. 0: Fiber cut.
  1: Loss of light at node B.
  3=1+2: Node B detects loss of light.
  3=3+0: Node B transmits OMS-FDI to node E, and OTS-BDI to node A.
  6=3+3: OTS-BDI received by node A, if the fiber from B to A is not cut.
  12=3+3+3+3: OMS-FDI from B received by E.
  12=12+0: OCh-FDI transmitted by E for all lightpaths passing through it, for example, those destined for G. 15=12+3: OCh-FDI received from E for lightpaths affected by the cut.
  2003=3+2000: Alarm raised by node B.

There is not much difference between the two methods.

8.4

- In an OXC with an electrical core and OEO conversion, the OXC can either use some of SONET/SDH overhead bytes, or use a digital wrapper, or an out-of-band signaling channel. The out-of-band channel can be carried on a separate wavelength, part of a wavelength (example: an OC-3 multiplexed into an OC-192 stream by the OLTs) or on a separate network, It can thus communicate in-band or out-of-band with other OXCs. It could monitor virtually all performance parameters used by SONET/SDH systems, including BER.
- In an OXC with an optical core and no OEO conversion, the OXC has to use an out-of-band signaling channel, carried as stated above. It could monitor a limited set of performance parameters such as optical power level and optical SNR. Direct monitoring of performance parameters such as BER would not be possible.

8.5

(a) Note that both τ and $\tau' > 2d_{\text{prop}}$ for the protocol to work.
(b) The time taken is always $\tau + \tau' + \tau = 2\tau + \tau'$.

9

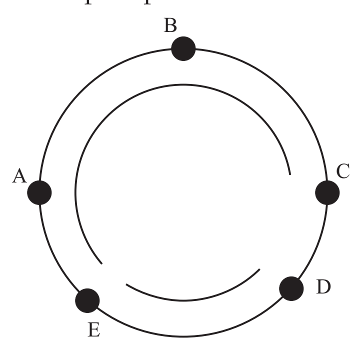
chapter

Network Survivability

9.1 Consider connection CE in the figure below. If link BC fails, we have the following

(a) path protection: Connection is restored along CDE (2 hops).

(b) line protection: Connection is restored along CDEABAE, which is very inefficient, compared to path protection.



Next consider a 1 hop connection DE. If link DE fails, both path and line protection use DCBAE to restore the connection. In this case, both need the same amount of bandwidth for restoration. In general, path protection is better (more efficient use of bandwidth) at restoring multihop connections than line protection.

9.2 Consider a link carrying traffic equal to its working capacity. If that link fails, then there is no way to restore traffic unless protection capacity = working capacity.

9.3 Note first that if both types of rings operate at, say, OC-12 speeds, the maximum concatenated connection stream that can be carried in a UPSR is OC-12c, whereas in a BLSR/2, it is OC-6c (because half the bandwidth on each fiber is reserved for protection). This is true regardless of the traffic pattern.

Consider rings with N nodes and an additional hub. Let t_i denote the traffic between node i and the hub. First note that since all traffic must be routed to the hub, the working capacity into the hub is only C, where C is the link speed. Therefore traffic patterns for which $\sum_{i=1}^{N} t_i > C$ cannot be supported,

We will show that in both the UPSR and the BLSR/2, all traffic patterns such that $\sum_{i=1}^{N} t_i \leq C$, can be supported (assuming traffic from a single node in a BLSR/2 can be split across two routes, if necessary). First consider the UPSR. Traffic from node i uses capacity t_i on every link in the ring (considering both working and protection traffic). Therefore this traffic can be supported provided $\sum_{i=1}^{N} t_i \leq C$.

Now consider the BLSR/2. Note that only a capacity of $C/2$ on every link is available for working traffic. Consider a traffic pattern such that $\sum_{i=1}^{N} t_i \leq C$. From node i, we route $t_i/2$ units clockwise and $t_i/2$ units counterclockwise on the ring to the hub. With this routing the traffic load on each link is $\sum_{i=1}^{N} t_i/2 \leq C/2$. Therefore this traffic pattern can be supported.

Therefore the UPSR and BLSR/2 can support the same set of traffic patterns in this case.

Thus a UPSR is superior for this application because it has the same traffic carrying capacity as a BLSR/2, and in addition,

- supports OC-12c connections,
- has faster protection, and,
- is a simpler and less expensive system.

9.4 The traffic distribution has all traffic between adjacent pairs of nodes. So the capacity is NC, where C is the bit rate on the fiber and N the number of nodes.

9.5 For the uniform traffic case, the average hop length is approximately $N/4$, where N is the number of nodes. So the reuse factor is approximately 4. So the capacity is $4C$, where C is the bit rate on the fiber.

9.6 (a) Left to the reader.

(b) Left to the reader.

(c) For UPSR, both the routes around the ring need to be used for work and protect. Thus each demand utilizes the bandwidth on every link in the network. Since the total demand is 80 STS-1s, this bandwidth is used by UPSR on every link in the network.

For BLSR, use the shortest path between nodes. This yields a load of 24 on the links A–B, B–C and C—D, a load of 8 on D–E and 22 on E–A. The average load arising from shortest path routing is a lower bound on the maximum load (from (8.10) together with the solution of Problem 8.7). Thus, under any routing scheme, the maximum load cannot be lower than $\lceil (24 \times 3 + 8 + 22)/5 \rceil = 21$. We can get a better lower bound by reasoning as follows. Club the nodes D and E into one node "DE" to get a 4-node ring with the following demand matrix (ignoring the demand between D and E).

STS-1	B	C	DE
A	12	6	16
B		8	16
C			14

The average link load (rounded up) due to shortest path routing on this 4-node ring is $\lceil (12 + 6 \times 2 + 16 + 8 + 16 \times 2 + 14)/4 \rceil = 24$. This is a lower bound on the maximum load for the original 5-node ring. (To prove this, observe that if this is not the case and there is a routing scheme for the 5-node ring which yields a better maximum load, then the same scheme can be applied to the 4-node ring leading to a contradiction.) Thus the maximum load of 24 obtained using shortest path routing is optimal.

(d) UPSR requires an OC-192 ring whereas BLSR only requires an OC-48 ring.

(e) BLSR is better since OC-48 rings are cheaper than OC-192 rings.

9.7 With 2 cuts, the network is partitioned into two clusters of nodes without any connection between the two clusters. Nodes within each cluster can communicate. Note that this is the case with all rings in general.

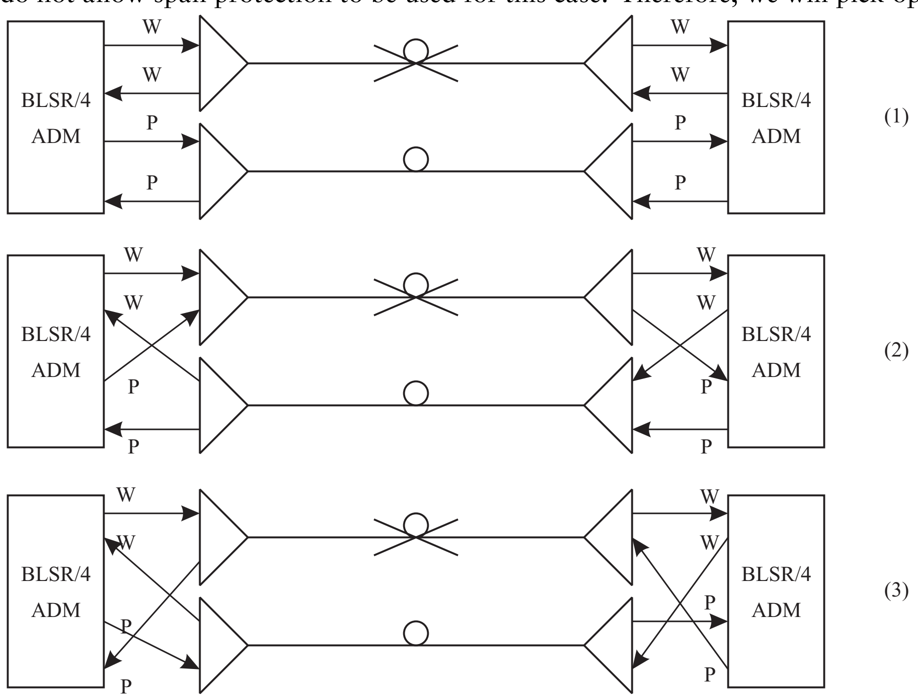
The UPSR can handle multiple cuts in one of the two rings because the other ring will be still fully functional. While it is quite likely that both fibers on a link get cut at the same time, this capability still enables the UPSR to continue providing service when a transmitter or receiver fails.

Unlike the UPSR, the BLSR/2 cannot handle multiple cuts because the protection capacity is shared.

The BLSR/4 can handle multiple failures of transmitter/receivers (one per span). It can handle simultaneous cuts of 1 fiber pair per span. Note that once span protection is used, line protection cannot be used any more to recover from another failure.

9.8 This scheme works fine under normal operation but cannot protect individual connections in case of a failure. For example, in Figure 9.4, if AB is cut, then receiver D must receive connections from A on the counter-clockwise ring but connections from B and C on the clockwise ring.

9.9 The three approaches are illustrated in the figure below. There is no difference between them as far as line protection is concerned. Also, span protection in the case of equipment failures works the same way in all the approaches. However span protection in the case of fiber cuts works differently. Option (1) allows span protection to be used in case of a single fiber cut, whereas options (2) and (3) do not allow span protection to be used for this case. Therefore, we will pick option (1).



9.10 (a) Once span protection is invoked, network management must prevent line protection from being invoked. Likewise, when line protection is invoked, network management must prevent span protection from being activated.

(b) Network management must allow span protection to be invoked on multiple spans, if needed.

9.11 As with a UPSR, this arrangement can handle multiple failures of fibers in one direction of the ring. This arrangement can also handle the fiber pairs AD and BC failing simultaneously.

9.12 Assume the nodes are located in the ring in the order C, A, H, B, D, that is, nodes A and B are at distance 1 from the hub H, and nodes C and D are at distance 2. Using shortest paths from each of the access nodes to the hub node, we need 2 units of working capacity on each of the links A–H and B–H, and one unit of working capacity on each of the links C–A and D–B, for a total of 6 units on all the links.

First consider OCh-DPRing. Assume each of the four work paths are assigned distinct wavelengths. Choose the protect paths as the longer paths on the ring between the access nodes and the hub. The protect path for each access node can be assigned the same wavelength as the work path (since all wavelengths are distinct). The work and protect paths from each of the nodes together consume one unit on every link in the ring, for a total of $4 \times 5 = 20$ units. Thus we need a protect capacity of $20 - 6 = 14$ units.

Next consider OCh-SPRing. We assume that while no wavelength conversion is allowed, the work and protect paths can have different wavelengths. (If this is not the case, and the work and protect paths must use the same wavelength, for example, if we have no transponders at the ends of the lightpaths, then the OCh-SPRing case is the same as the OCh-DPRing case and we would use one we dedicate one wavelength around the ring for each access node.) Assume that the work paths A–H and B–H are both assigned the wavelength λ_1 and the work paths C–A–H and D–B–H are both assigned the wavelength λ_2. We need to dedicate one wavelength each to protect the traffic on wavelengths λ_1 and λ_2. Thus we need a total protect capacity of 10 units.

Wavelength conversion would not change the answer in both the cases. Wavelength conversion or not, with OCh-DPRing each access node needs capacity on every link in the ring. Similarly, even with wavelength conversion, the work traffic would use 2 units of capacity on some link so that 2 units of protect capacity across the ring would be needed.

9.13 Left to the reader.

10 chapter

WDM Network Design

10.1 Note that the topology seen by the routers is the lightpath topology of Figure 10.2(b) with a capacity of x lightpaths on links A–B and B–C, and a capacity of y lightpaths on link A–C. Denote the A–B traffic by α, the B–C traffic by β, and the A–C traffic by γ. The traffic matrices or, equivalently, values of α, β and γ that can be supported depend on the constraints, if any, imposed by routing.

First, assume that all traffic is routed on the direct path in the lightpath topology. This would be the case if load-balancing on alternate paths is not supported by the IP layer routing protocol. In this case, the allowed values of α, β, and γ are those that satisfy $\alpha \leq x$, $\beta \leq x$, and $\gamma \leq y$.

If alternate routing is allowed, the answer is much more complicated. Let α_1 denote the A–B traffic routed on the direct path A–B, and α_2 the A–B traffic routed through C, that is, on the path A–C–B in the lightpath topology. Similarly, define β_1, β_2, γ_1 and γ_2. Note that the traffic γ_2 is dropped to the IP router at node B and reinserted by it, whereas the traffic γ_1 passes through node B without touching the IP router at node B. Then, the supported values of α, β, and γ are those for which the following inequalities has a feasible solution.

$$\alpha_1 + \beta_2 + \gamma_2 \leq x$$

$$\alpha_2 + \beta_1 + \gamma_2 \leq x$$

$$\alpha_2 + \beta_2 + \gamma_1 \leq y$$

10.2 (a) The routing and wavelength assignment is as follows:

Traffic stream	Wavelength	Path
AB	$\lambda_1, \lambda_2, \lambda_3$	AB
AD	$\lambda_1, \lambda_2, \lambda_3$	AD
BC	λ_1, λ_2	BC
CD	λ_1, λ_2	CD
BD	λ_3, λ_4	BCD
BD	λ_4	BAD

(b) The minimum total traffic load due to all the connections can be computed by using the minimum number of hops required for each connection as follows:

Traffic stream	Traffic	Min. hops	Traffic load
AB	3	1	3
AD	3	1	3
BC	2	1	2
BD	3	2	6
CD	2	1	2
Total			16

Since there are only 4 edges to carry this load, the average load per edge is $16/4 = 4$, and the maximum load per edge is therefore at least 4. Thus, at least 4 wavelengths are required.

(c) Node A needs 3 ADMs, node B 4, node C 2, and node D, 4 ADMs.

(d) Each node would need 4 ADMs.

10.3 Consider any source node. The $N-1$ traffic streams from that node to the other nodes, when routed on their shortest paths take up a total number of hops of

$$h_{\text{odd}} = 2\left(1+2+3+\cdots+\frac{N-1}{2}\right) = \frac{N^2-1}{4}$$

for odd N and

$$h_{\text{even}} = 2\left(1+2+3+\cdots+\frac{N}{2}-1\right) = \frac{N^2}{4}$$

for even N. The traffic between each pair of nodes is $t/(N-1)$, and so the average load due to this traffic on each edge is

$$\frac{h_{\text{odd}}\frac{t}{N-1}N}{2N} = \frac{N+1}{8}t$$

for odd N and

$$\frac{h_{\text{even}}\frac{t}{N-1}N}{2N} = \frac{N+1+\frac{1}{N-1}}{8}t$$

for even N.

10.4 Since two adjacent nodes use different paths along the ring, only $N/2$ nodes use any given edge on the ring. But any node using this edge routes $\lceil t \rceil$ lightpaths through it and uses $\lceil t \rceil$ different wavelengths. Thus,

$$W = \frac{N}{2}\lceil t \rceil$$

for even values of N.

For odd values of N, the node diametrically opposite the hub routes $\lceil t/2 \rceil$ lightpaths in one direction and $\lceil t/2 \rceil$ lightpaths in the other direction. Therefore,

$$W = \frac{N-1}{2}\lceil t \rceil + \left\lceil \frac{t}{2} \right\rceil$$

for odd values of N.

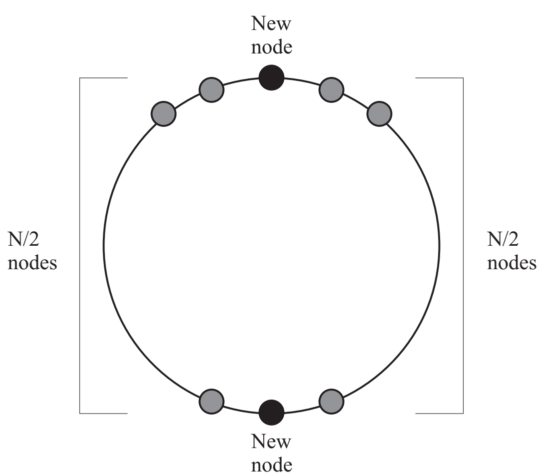
10.5 For $N = 2$ we require only 1 wavelength and

$$W(2) = \frac{2^2}{8} + \frac{2}{4} = 1.$$

Suppose

$$W(N) = \frac{N^2}{8} + \frac{N}{4}$$

wavelengths are sufficient for some $N \geq 2$, even. Then add 2 more nodes as shown below. Each of the new nodes uses the shortest path to communicate with the other N nodes and shortest paths (clockwise or counterclockwise) to communicate with each other. The number of additional wavelengths needed is $\frac{N}{2} + 1$.



By the induction hypothesis, the number of wavelengths required is

$$\frac{N^2}{8} + \frac{N}{4} + \frac{N}{2} + 1 = \frac{(N+2)^2}{8} + \frac{N + 2N + 4 - 2N - 2}{4} = \frac{(N+2)^2}{8} + \frac{N+2}{4}.$$

10.6 We will first solve the problem for the case of 1 lightpath between each pair of nodes. When $N = 3$, we require only 1 wavelength and

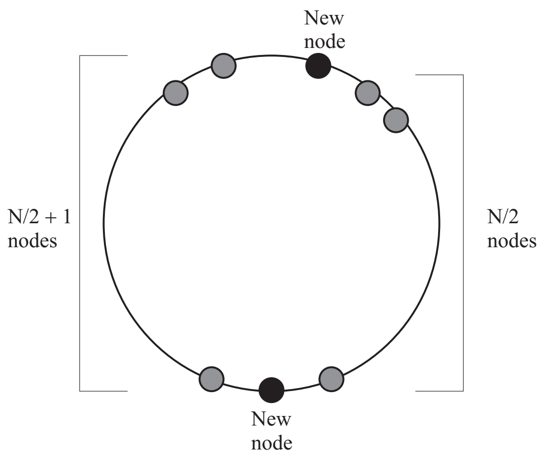
$$W(3) = \frac{3^2 - 1}{8} = 1.$$

Suppose

$$W(N) = \frac{N^2 - 1}{8}$$

wavelengths are sufficient for some $N \geq 3$, odd.

Add 2 more nodes as shown below. The routing is the same as in the N-even case and the new nodes use the shortest path to communicate with each other. This requires $\frac{N-1}{2} + 1$ additional wavelengths.



By the induction hypothesis, the number of wavelengths required is

$$\frac{N^2 - 1}{8} + \frac{N - 1}{2} + 1 = \frac{(N + 2)^2 - 1}{8}.$$

When we need $\lceil t/(N-1) \rceil$ lightpaths between each pair of nodes, the expression above must be modified to

$$W(N) = \left\lceil \frac{t}{N-1} \right\rceil \left(\frac{N^2 - 1}{8} \right).$$

10.7 For N even, we have

$$\begin{aligned} \sum_{j=1}^{N} h_{ij} &= 2\left[1 + 2 + \cdots + \left(\frac{N}{2} - 1\right)\right] + \frac{N}{2} \\ &= \left(\frac{N}{2} - 1\right)\frac{N}{2} + \frac{N}{2} = \frac{N^2}{4}. \end{aligned}$$

Thus

$$H_{\min} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij}}{N(N-1)} = \frac{N^2}{4(N-1)} = \frac{N+1}{4} + \frac{1}{4(N-1)}.$$

For N odd,

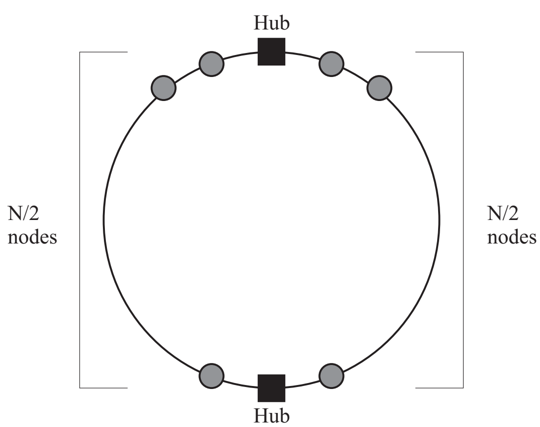
$$\sum_{j=1}^{N} h_{ij} = 2\left(1+2+\cdots+\frac{N-1}{2}\right) = \left(\frac{N-1}{2}\right)\left(\frac{N+1}{2}\right)$$
$$= \frac{N^2-1}{4}.$$

Therefore,

$$H_{\min} = \frac{\frac{N^2-1}{4}}{N-1} = \frac{N+1}{4}.$$

10.8 Consider the *double hub* architecture shown in the figure below. Traffic from each node is routed to the nearest hub through lightpaths. Traffc between hubs (where the source node is closer to one hub, and the destination node to the other) is routed through lightpath setup between them.

Since each node has to route t units of traffic, it sets up sets up $\lceil t \rceil$ lightpaths to the closest hub. These lightpaths require a total of $2N\lceil t \rceil$ LTs in the network. We assume the number of nodes $N = 4k$. These lightpaths require $k\lceil t \rceil$ wavelengths—each of the k nodes on one side (left or right) that is closest to a given hub, uses $\lceil t \rceil$ distinct wavelengths. The same set of wavelengths can be reused in the four quadrants of the ring.



Assume a node is closer to hub 1 than hub 2. Traffic from this node to the $2k$ nodes that are closer to hub 2 has to be routed on the lightpaths between the hubs. This traffic amounts to $\frac{t}{N-1}$ for each pair of nodes where one is closer to hub 1, and the other to hub 2. Since there are $4k^2$ such node pairs, the total traffic that is to be routed between the two hubs is $4k^2\frac{t}{N-1}$. Assume half this traffic is routed clockwise and the other half, counterclockwise. This traffic thus requires requires $4\lceil 2k^2\frac{t}{N-1}\rceil$ LTs, and $\lceil 2k^2\frac{t}{N-1}\rceil$ wavelengths.

Putting all this together, the number of LTs required per node in this architecture is $2\lceil t \rceil + \frac{4}{N}\lceil \frac{N^2}{8}\frac{t}{N-1}\rceil$, and the number of wavelengths is $\frac{N}{4}\lceil t \rceil + \lceil \frac{N^2}{8}\frac{t}{N-1}\rceil$. In comparison, the single hub architecture requires $2\lceil t \rceil$ LTs and $\frac{N}{2}\lceil t \rceil$ wavelengths. Thus the double hub architecure requires more LTs but fewer wavelengths than the single hub architecture.

10.9 First, there is a typographical error in the problem statement, where $\lambda^{s,t}$ should be λ^{sd}.

Since the traffic is bidirectional, $\lambda^{sd} = \lambda^{ds}$.

The objective function changes to $\sum_{i<j} c_{ij} \cdot b_{ij}$, where condition $i < j$ is due to the fact that the lightpaths are bidirectional. The condition ensures that we consider a lightpath only once in the summation.

The total flow for all pairs (i, j) is $\lambda_{ij} = \sum_{s,d} \lambda_{ij}^{sd}$, and that $\lambda_{ij} \le r$.

The degree constraints and bidirectional lightpath constraints remain the same. The nonnegativity and integer constraints remain the same except variable $\lambda_{\max}$ is not considered.

10.10 The network of Figure 10.21(a) is much better than that of Figure 10.21(b). Consider a unidirectional lightpath from B to C. The network of Figure 10.21(b) cannot support it, but (a) can. Note that there is no way around this problem. If we reverse the directions of wavelengths on the link between C and the hub, then we cannot support a connection from C to A.

10.11 In the multifiber network (A), label the fibers from 1 to P and and wavelengths from 1 to W. In the single fiber-pair network (B), label wavelengths from 1 to PW. We will associate wavelength (i, w) in network A (i represents the fiber index and w the wavelength on that fiber) with wavelength $(i-1)W + w$ in network B.

Consider a lightpath in network A that uses (i_1, w_1) on one link and (i_2, w_1) on the next link. Note that the wavelength must be the same as there is no conversion in the network. An equivalent lightpath in network B uses wavelength $W(i_1 - 1) + w_1$ and $W(i_2 - 1) + w_1$ on the same links. Note that this is always feasible because of degree P wavelength conversion in network B, which implies that a wavelength $W(i-1) + w$ can be converted to any wavelength $W * +w$ on the next link. Here $*$ denotes any of the P possible values of $i - 1$. Therefore network B can support any lightpath supported by network A.

The proof in the reverse direction is similar.

10.12 The network has approximately $O(n)$ rows and $O(n)$ columns. Thus $D \approx n$ and $M \approx n^2$. We can do the routing so that L is a constant. Therefore both $(L-1)D + 1$ and $L\sqrt{M} - L + 2$ are $O(n)$, which is the number of wavelengths required, since each node pair requires a separate wavelength to communicate in this example.

10.13 Suppose there are $K(x)$ lightpaths of length $\ge x$ hops. The average load due to these lightpaths, say $l(x)$, satisfies

$$\frac{xK(x)}{M} \le l(x) \le L$$

so that $K \le LM/x$. Assign LM/x separate wavelengths to these lightpaths. Next consider the lightpaths of length $\le x - 1$ hops. Each of these intersects with at most $(L-1)(x-1)$ other such lightpaths, and so will need at most $(L-1)(x-1) + 1$ additional wavelengths. So we have

$$W \le LM/x + (L-1)(x-1) + 1$$

for every x. The minimum of the RHS occurs for $x = \sqrt{LM/(L-1)}$. For large L, the minimum occurs for $x \approx \sqrt{M}$ which corresponds to the case considered in the text.

10.14 For a two node network, the algorithm clearly uses only L wavelengths. Consider a network with n nodes and maximum load L. Consider the $(n-1)$ node network obtained by deleting node n and terminating all lightpaths that would have terminated at node n, at node $(n-1)$. This network has load at most L, and by the induction hypothesis the greedy algorithm uses at most L wavelengths for this network. Now consider the n node network. The lightpaths terminating at node n can keep the same color that they were assigned in the $(n-1)$ node network; no conflicts occur since these lightpaths share both the edge from $(n-2)$ and $(n-1)$ and the edge from $(n-1)$ to n and no conflicts occur on the edge from $(n-2)$ to $(n-1)$. Suppose there are x such lightpaths, which take up x wavelengths on the edge from $n-2$ to $n-1$. Then the lightpaths from node $(n-1)$ to n are at most $L-x$ in number since the load is L. Therefore the greedy algorithm can assign the $L-x$ remaining wavelengths to these lightpaths, and thus uses no more than L colors in all.

10.15 The above proof holds, except that in the last step, note that any algorithm that choses any available color from a *fixed* set of L colors never runs out of a color.

10.16 The construction is as follows. Number the nodes in the ring starting at an arbitrary node 0, and proceeding counterclockwise up to node $N-1$. Define the following set of $2L-2$ lightpaths, all proceeding counter clockwise along the ring between the two nodes listed below:

$$a_1 = [0, \tfrac{N}{2}], a_2 = [1, \tfrac{N}{2}+1], ..., a_{L-1} = [L-2, \tfrac{N}{2}+L-2],$$

$$b_1 = [\tfrac{N}{2}, 1], b_2 = [\tfrac{N}{2}+1, 2], ..., b_{L-1} = [\tfrac{N}{2}+L-2, L-1].$$

Note that all the a_i overlap on edges between nodes $L-2$ and $\frac{N}{2}$, all the b_i overlap on edges between nodes $\frac{N}{2}+L-2$ and 1, and each a_i overlaps with each b_i. Thus all of them must be assigned separate wavelengths. The load can be seen to be L.

Now add an additional lightpath

$$c = [\tfrac{N}{2}-1, \tfrac{N}{2}+L-1].$$

Note that c overlaps with all the a_i and b_i and that the load is still L. Therefore $2L-1$ wavelengths are required to support these $2L-1$ lightpaths.

Note that for the construction to work, we must have

$$\tfrac{N}{2}+L-2 \le N-1 \text{ or } N > 2L-1.$$

10.17 First consider the case when N is odd. Since $(N^2-1)/8$ is an integer when N is odd, the fully optical network of Example 10.4 uses $(N^2-1)/8$ wavelengths to support this traffic $(t = N-1)$, without wavelength conversion. (See Problem 10.6.) Thus $(N^2-1)/8$ wavelengths are sufficient to support this traffic, with or without wavelength conversion, when N is odd. From the solution to Problem 10.7 (with $t = N-1$) and (10.10), the average load on each edge is $(N^2-1)/8$. Thus $(N^2-1)/8$ wavelengths are also necessary in this case, with or without conversion.

Now consider the case when N is even. Using the fully optical network of Example 10.4, $(N^2+2N)/8$ wavelengths are sufficient to support this traffic with no wavelength conversion. From (10.10) with $t = N-1$, the average load on each edge is $N^2/8$. Thus $\lceil N^2/8 \rceil$ wavelengths are necessary to support this traffic.

Consider the case with full wavelength conversion where N is even. We give a construction below that has a maximum load of

$$\frac{N^2}{8} + \frac{1}{2}$$

when N is a multiple of 4 and

$$\frac{N^2}{8} + 1$$

when N is not a multiple of 4. With full wavelength conversion, these also correspond to the number of wavelengths that are sufficient to support this traffic. The construction works as follows. Consider all 1-hop lightpaths between node pairs. These can be supported with a load of 1 by routing the lightpaths along the shortest path between the nodes. Similarly, for $k < N/2$, k-hop lightpaths can be supported with a load of k by routing them along the shortest paths. Thus the total load due to all lightpaths of length $< N/2$ is

$$1 + 2 + \ldots + \frac{N}{2} - 1 = \frac{N^2}{8} - \frac{N}{4}.$$

The only remaining lightpaths are the lightpaths between nodes that are diametrically opposite in the ring, that is, those that are $N/2$ hops apart in the ring. For these lightpaths we have two choices of routes. The routing is done as follows. The lightpath that starts at node 0 is routed clockwise along the ring. The lightpath that starts at node 1 is routed counter-clockwise, the one that starts at node 2 is routed clockwise, and so on. The reader can verify that this routing induces a load of

$$\frac{N}{4} + 1$$

when N is a multiple of 4 and

$$\frac{N+2}{4}$$

when N is not a multiple of 4.

Thus considering all the lightpaths, the maximum load of this construction is

$$\frac{N^2}{8} + 1$$

when N is a multiple of 4 and

$$\frac{N^2}{8} + \frac{1}{2}$$

when N is not a multiple of 4.

Observe therefore, that having wavelength conversion helps us to reduce the number of wavelengths in this case. The overall results are summarized below:

	No conversion		Full conversion	
	Necessary	Sufficient	Necessary	Sufficient
N odd	$\frac{N^2-1}{8}$	$\frac{N^2-1}{8}$	$\frac{N^2-1}{8}$	$\frac{N^2-1}{8}$
N even	$\left\lceil \frac{N^2}{8} \right\rceil$	$\frac{N^2}{8} + \frac{N}{4}$	$\left\lceil \frac{N^2}{8} \right\rceil$	$\frac{N^2}{8} + 1,\ N = 4m$
				$\frac{N^2}{8} + \frac{1}{2},\ N = 4m + 2$

10.18 Consider a 3-node star network with one lightpath between every pair of nodes. The maximum load $L = 2$ but $\frac{3}{2}L = 3$ wavelengths are necessary to perform the wavelength assignment. To see this, observe that each lightpath shares an edge with the other two so that all three lightpaths must be assigned distinct wavelengths.

10.19 Consider a ring network with load L. Cut it at any node, say node Z, to obtain a line network. The lightpaths in this line network can be colored with $W \leq L$ wavelengths, since the maximum load is L. However lightpaths passing through node Z are split into two (sub) lightpaths in the line network and the two (sub) lightpaths may be assigned different colors. Say there are k such lightpaths with colors $(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)$ assigned to their two parts in the line network.

By using full wavelength conversion at node Z, wavelengths x_i can be converted to y_i and vice versa, $i = 1, 2, \ldots, k$. This allows the network to support all lightpath requests with load $L \leq W$.

10.20 From Lemma 10.7 we have

$$W(N, L) \leq L + W\left(\frac{N}{2}, L\right) .$$

Add dummy nodes to the line network so that N is a power of 2. Then,

$$\begin{aligned} W(N, L) &\leq L + W\left(\frac{N}{2}, L\right) \\ &\leq L + L + W\left(\frac{N}{4}, L\right) \\ &\leq L + L + L + W\left(\frac{N}{8}, L\right) \\ &\quad \ldots \\ &\leq (\log_2 N - 1)L + W(2, L). \end{aligned}$$

Since $W(2, L) = L$, we have

$$W(N, L) \leq (\log_2 N)L.$$

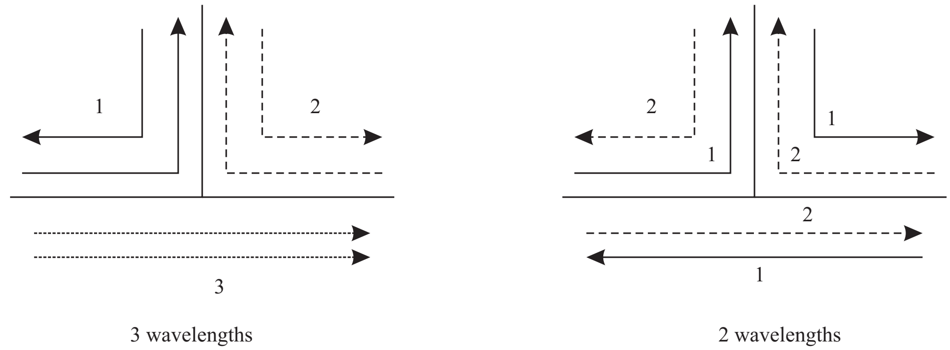
If N is not a power of 2, $W(N, L) \leq \lceil \log_2 N \rceil L$.

The algorithm is as follows: Divide the $\lceil \log_2 N \rceil L = kL$ wavelengths into k groups of L wavelengths each. The nodes are indexed using $\lceil \log_2 N \rceil$-bit binary numbers, say $x_1, x_2, \ldots x_k$ where $k = \lceil \log_2 N \rceil$. Given a lightpath from node $x = (x_1, x_2, \ldots x_k)$ to node $y = (y_1, y_2, \ldots y_k)$, find the least index i for which $x_i \neq y_i$. Use any available wavelength from group i for this lightpath. The pseudocode for the algorithm is given below:

for $(i = 1; i \leq k; i++)$ if $(x_i \neq y_i)$ break;
for $(w = (i-1)L; w < iL; w++)$ if w is available, break;
assign w to the lightpath.

10.21 Cut the ring network at any node. For lightpaths not passing through the cut node, say node Z, $\lceil \log_2 N \rceil L$ wavelengths suffice since we can use the online wavelength assignment algorithm on the resulting line network. Allocate L additional wavelengths for lightpaths passing through node Z and assign any available wavelength from this set to a lightpath passing through node Z. We never run out of wavelengths since the maximum load $= L$ means no more than L lightpaths pass through node Z.

10.22



10.23 In a network using full wavelength conversion, a lightpath request is blocked if there is no free wavelength on some link in the path. The probability that no wavelength is free on any given link is π^W. So the probability that there is no blocking on any of the H hops, using the link independent property, is given by $= (1-\pi^W)^H$. Therefore,

$$P_{\mathrm{b,fc}} = 1 - (1-\pi^W)^H.$$

10.24 We have

$$\pi_{\mathrm{nc}} = 1 - \left(1 - P_{\mathrm{b,nc}}^{1/W}\right)^{1/H}.$$

For small $P_{\mathrm{b,nc}}^{1/W}$ (small $P_{\mathrm{b,nc}}$ and W not large), using $(1-x)^n \approx 1 - nx$, for small x,

$$\pi_{\mathrm{nc}} = \frac{P_{\mathrm{b,nc}}^{1/W}}{H}.$$

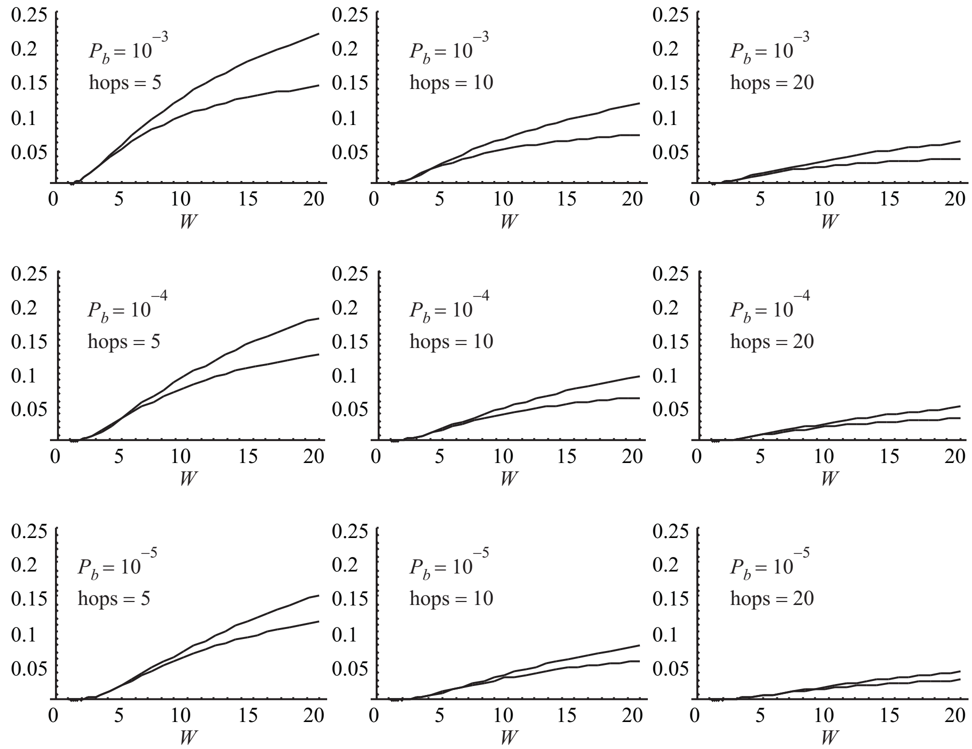
Also,

$$\pi_{\mathrm{fc}} = \left(1 - (1 - P_{\mathrm{b,fc}})^{1/H}\right)^{1/W}.$$

Again, using $(1-x)^n \approx 1 - nx$ for small x, for small $P_{\mathrm{b,fc}}$, we get

$$\pi_{\mathrm{fc}} \approx \left(\frac{P_{\mathrm{b,fc}}}{H}\right)^{1/W}.$$

The exact expression and the approximation for π_{nc} are plotted versus the number of wavelengths W, for various values of P_b and number of hops, in the plots below. The approximation consistently underestimates the utilization so that the lower curve in each plot corresponds to the approximation. It can be seen that the approximation is accurate only for $W \leq 5$ or so, when $P_b = 10^{-3}$. When $P_b = 10^{-5}$, the range of accuracy of the approximation increases to around $W \leq 10$.



The approximation for $P_{\mathrm{b,fc}}$ is so accurate for $P_b \leq 10^{-3}$ that the curves for the approximate and exact expressions are indistinguishable. Hence these curves are not shown here.

10.25 The probability that a wavelength is free on link k, given that it is free on links $1, 2, \ldots, k-1$, is given by $1 - \pi_n$, by the definition of π_n. (It only matters that it is free on $k-1$.) Therefore, the probability that a wavelength λ is free on link 1 is $1 - \pi_n$. The probability that it is free on links 1 and 2 is $(1-\pi_n)^2$. The probability that it is free on all H links is $(1-\pi_n)^H$. So the probability that wavelength λ is not free is given by $1 - (1-\pi_n)^H$. Thus,

$$P_{\mathrm{b,nc}} = \left[1 - (1-\pi_n)^H\right]^W .$$

10.26 (a)

Gb/s	B	C	D	E
A	2	3	1	2
B		1	4	2
C			2	3
D				1

(b) If we route each lightpath along its shortest path, starting from the top left of the matrix above, and going down row by row, and assigning the lowest possible wavelength to each lightpath, we get the following assignment:

Lightpath	Wavelength
AB	1
AB	2
AC	3
AC	4
AC	5
AD	1
AE	2
AE	3
BC	1
BD	1
BD	2
BD	3
BD	4
BE	5
BE	6
CD	1
CD	2
CE	3
CE	4
CE	7
DE	2

(c) The most heavily loaded link is DE, with a total load of 7, which is also equal to the number of wavelengths.

Chapter 11

Access Networks

11.1 Broadcast-and-select PON, laser transmitter

Received power$= -14 - 10\log N \geq -40$. Therefore $N \approx 400$.

Broadcast-and-select PON, LED transmitter

Received power$= -31 - 10\log N \geq -40$. Therefore $N \approx 8$.

WDM PON

Same as a broadcast-and-select PON.

WRPON, laser transmitter

Received power$= -13 - L \geq -40$, where L is the router loss, which depends on N. For $N = 64$, $L = 12$ dB, which is still feasible here.

WRPON, LED spectral slicing

Here the router acts as a spectral slicer. Received power$= -31 - 10\log(2N) \geq -40$. Therefore $N \approx 4$.

11.2 Total bandwidth required = $20 \times 12 = 240$ Mb/s. This cannot be supported by a single transmitter. We could use two wavelengths (2 lasers) at the CO—one at 1.3 and the other at 1.5 μm. They would be combined using a 1.3/1.5 coupler, and sent through the AWG. The AWG, because of its periodicity, serves as a router for both the wavelengths. Each ONU would have a 1.3/1.5 coupler to select one of the wavelengths.

12 chapter

Photonic Packet Switching

12.1 Pulse 1 is delayed by every one of the k stages for a total delay of

$$(T-\tau)+2(T-\tau)+\cdots+2^{k-1}(T-\tau)=(2^k-1)(T-\tau).$$

Note that the pulses not delayed by stage j are those for which the binary representation of $(i-1)$ has a 1 in the jth bit (counting from right to left, starting from 1). Thus the total delay *not* undergone by pulse i is $(i-1)(T-\tau)$. Therefore, pulse i undergoes a delay of

$$(2^k-1-i+1)(T-\tau)=(2^k-i)(T-\tau).$$

Assume that pulse 1 occurs at time 0, pulse 2 occurs at time T, pulse i occurs at time $(i-1)T$, ..., at the input. Then, at the output, pulse i occurs at time

$$(i-1)T+(2^k-i)(T-\tau),$$

whereas pulse $(i-1)$ occurs at

$$(i-2)T+(2^k-i+1)(T-\tau).$$

The difference is

$$T-(T-\tau)=\tau\ .$$

Therefore, the pulses are τ apart at the output.

12.2 We can arrange the timing of the pulses such that pulse i is delayed by those stages in which $(i-1)$ has a 1 in its binary representation (counting from left to right starting from 1). Then pulse i undergoes a delay of $(i-1)(T-\tau)$. Thus pulse i occurs at the output at time

$$(i-1)T+(2^k-i)(T-\tau)+(i-1)(T-\tau)=(i-1)T+(2^k-1)(T-\tau).$$

Likewise, pulse $i-1$ occurs at the output at time $(i-2)T + (2^k-1)(T-\tau)$, and the difference between the two is T. Note that the switching time required is $\ll \tau$, which is not feasible for small τ.

12.3 Delay stage i, $i = 1, 2, \ldots k-1$, should be encountered if the binary representation of x has a 1 in position i, counting from left to right, starting from 1.

Let $b_1 b_2 \ldots b_{k-1}$ be the binary representation of x. Let $b_0 = 1$. Then we have the following truth table for c:

b_{i-1}	b_i	c_i
1	1	1
0	1	0
1	0	0
0	0	1

Thus $c_i = b_{i-1} \oplus b_i$, where $\oplus$ denotes the exclusive or (XOR) operation.

12.4 We know that the transfer function of a 3 dB coupler is

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}.$$

If E_i denotes the field of the input pulse, then the fields of the clockwise and counterclockwise pulses can be written as

$$\begin{pmatrix} E_c \\ E_{cc} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} E_i \\ jE_i \end{pmatrix}.$$

If a phase shift ϕ is introduced between them, then

$$\begin{pmatrix} E_c \\ E_{cc} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} E_i \, e^{i\phi} \\ jE_i \end{pmatrix}.$$

After the second pass through the coupler,

$$\begin{pmatrix} E_B \\ E_A \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix} \begin{pmatrix} E_i \, e^{i\phi} \\ jE_i \end{pmatrix}.$$

We have therefore,

$$E_B = \frac{1}{2}(e^{i\phi} - 1)E_i \; .$$

If $\phi = 0$, $E_B = 0$. For $|E_B| = |E_i|$, $e^{i\phi} = -1$ or $\phi = \pi$.

12.5

- The duration of the header is 80 bits at 1 Gb/s, that is, 80 ns. If the payload duration must be 90% of the overall packet duration, it must be 9 times the header duration, or $9 \times 80 = 720$ns. At 100 Gb/s, the payload needs to be $720 \times 100 = 72{,}000$ bits, or 9000 bytes, long.
- If the payload size must be limited to 1000 bytes, that is one-ninth, and the same efficiency maintained, the header must be transmitted 9 times faster, that is at 9 Gb/s.

- The header duration is 80 ns. The guard time effectively increases the header duration by 1000 ns to 1080 ns. To maintain an efficiency of 90%, the payload duration must be 9 times larger, that is 9720 ns. At 100 Gb/s, the payload is thus $9720 \times 100 = 972\,000$ bits, or 121,500 bytes, long.